1. A sample of 35 different payroll departments found that employees worked an average of 240.6 days a year. If the population standard deviation is 18.8 days, find the 90% confidence interval for the average number of days μ worked by all employees who are paid through payroll departments.

Given:

n = 35 $\bar{x} = 240.6$ $\sigma = 18.8$ 90% CI $Z_{\alpha/2} = 1.645$ $\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

 $240.6 \pm 1.645 \frac{18.8}{\sqrt{35}}$

 $240.6 \pm 5.23 \quad \Longrightarrow \ 235.4 \ < \ \mu < 245.8$



2. A study of 65 bolts of carpet showed that their average length was 78.2 yards. The standard deviation of the population is 2.6 yards. a) Find the best point estimate of the mean. b) Find the 80% confidence interval for the mean length per bolt of carpet?

Given:

n = 65 $\bar{x} = 78.2$ $\sigma = 2.6$ 80% CI $Z_{\alpha/2} = 1.28$ a) The best point estimate is the sample mean, 78.2 yards

b) $\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $78.2 \pm 1.28 \frac{2.6}{\sqrt{65}}$ $78.2 \pm 0.41 \implies 77.8 < \mu < 78.6$

CASIO 9750 F4 for INTR, F1 for Z, F1 for 1-S 1-Sample ZInterval Data :Variable C-Level :0.8 0 :2.6 2 :78.2 n :65 Save Res:None None LIST	TI84 CE Plus STAT, TESTS, then 7: Z-Interval ZInterval Inpt:Data Stats σ:2.6 x:78.2 n:65 C-Level:0.8
1-Sample ZInterval Left =77.786712 Right=78.613288 X =78.2 n =65	ZInterval (77.787,78.613) x=78.2 n=65

3. The average number of mosquitos caught in 36 mosquito traps in a particular environment was 800 per trap. The standard deviation of mosquitos caught in the entire population of traps is 100 mosquitos. What is the 99% confidence interval for the true mean number of mosquitos caught in all mosquito traps?

Given:

 $n = 36 \quad \bar{x} = 800 \quad \sigma = 100 \quad 99\% \ CI \quad Z_{\alpha/2} = 2.576$ $\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $800 \pm 2.576 \frac{100}{\sqrt{36}}$ $800 \pm 42.93 \quad = \Rightarrow 757 < \mu < 843$



4. The average number of mosquitos caught in 36 mosquito traps in a particular environment was 800 per trap. The standard deviation of mosquitos caught in the entire population of traps is 100 mosquitos. What is the 95% confidence interval for the true mean number of mosquitos caught in all mosquito traps? Compare the result to the previous question. Why this interval is smaller?

n = 36 $\bar{x} = 800$ $\sigma = 100$ 95% CI $Z_{\alpha/2} = 1.96$

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$800 \pm 1.96 \frac{100}{\sqrt{36}}$$

$$800 \pm 32.7 \implies 767 < \mu < 833$$

The 95% confidence is smaller since there is less of a chance that the mean is contained in the interval as opposed to the 99% confidence interval.

CASIO 9750 F4 for INTR, F1 for Z, F1 for 1-S 1-Sample ZInterval Data :Variable C-Level :0.95 0 :100 X :800 n :36 Save Res:None	TI84 CE Plus STAT, TESTS, then 7: Z-Interval ZInterval Inpt:Data Stats σ:100 x:800 n:36 C-Level:0.95
1-Sample ZInterval Left =767.333934 Right=832.666066 X =800 n =36	ZInterval (767.33,832.67) x=800 n=36

5. Find $t_{\alpha/2}$ for n = 20 and a 95% confidence interval.

df = n - 1 = 19						
Table F	The t Distribution					
	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845



6. Find the critical value $t_{\alpha/2}$ needed to construct a confidence interval of the given level with the given sample size. Level 90%, sample size 8

Table F	The t Distribution					
	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355

$\begin{array}{l} CASIO \ 9750\\ F5 \ for \ DIST, \ F2 \ for \ T, \ F3 \ for \ InvT\\ Area = \alpha/2 = 0.10/2\\ \hline Inverse \ Student-t\\ Data & $Variable\\ Area & $0.10{\div}2\\ df & 17\\ Save \ Res$None\\ Execute \\ \end{array}$	TI84 CE 2 nd Distr, then 4: invT Area = α/2 = 0.10/2 invT area:0.10/2 df:7 Paste Enter invT(0.10/2,7)
EXE	-1.894578584
Inverse Student-t xInv =1.89457861	

Five squirrels were found to have an average weight of 8.9 ounces with a sample standard deviation is 0.9.
 Find the 95% confidence interval of the true mean weight.

Given: n = 5 $\bar{x} = 8.9$ s = 0.9 95% CI $t_{\alpha/2} = 2.776$ df = 4 $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ $8.9 \pm 2.776 \frac{0.9}{\sqrt{5}}$ 8.9 ± 1.11 $(7.8, 10.0) ==> 7.8 < \mu < 10.0$



Boxes of raisins are labeled as containing 22 ounces. Following are the weights, in ounces, of a sample of 12 boxes. It is reasonable to assume that the population is approximately normal.
 22.08, 22.25, 21.95, 22.39, 22.08, 22.11, 21.89, 21.60, 22.34, 22.03, 22.06, 22.32

Construct a 90% confidence interval for the mean weight.

Given the raw data, if using the formula to calculate the interval you need to find the sample mean & sample standard deviation.

Sample mean, $\bar{x} = 22.09$ sample standard deviation, s = 0.22 n = 12 (twelve data values). 90%, for df = 11, $t_{\alpha/2} = 1.796$ $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ $22.09 \pm 1.796 \frac{0.22}{\sqrt{12}}$ 22.09 ± 0.1141 $21.977 < \mu < 22.206$

<i>CASIO</i> 9750	TI84 CE Plus		
First, enter data on List 1	First, enter data on List 1		
F4 for INTR, F2 for T, F1 for 1-S then	STAT, TESTS, then 8: T-Interval		
press F1 to change Data to List	then for Inpt select Data,		
1-Sample tInterval	press Enter:		
Data :List C-Level :0.9	TInterval		
List List1	Inpt:Data Stats		
Freg :1 Sava Pas:None	List:L1 Enog:1		
Execute	C-Level:0.9		
CALC	Calculate		
1 Comple + Internal	Tisterus		
laft =21 9772526	(21, 977, 22, 206)		
Right=22.2060807	$\bar{x}=22.09166667$		
z =22.0916667	Sx=0.2206944961		
sx =0.22069449	n=12		
n =12			

Six measurements were made of the magnesium ion concentration (in parts per million, or ppm) in a city's municipal water supply, with the following results. It is reasonable to assume that the population is approximately normal. 180, 176, 182, 182, 216, 204
 Calculate the 99% confidence interval for the mean magnesium ion concentration.

Given the raw data, if using the formula to calculate the interval you need to find the sample mean & sample standard deviation.

Sample mean, $\bar{x} = 190$ sample standard deviation, s = 16.1n = 6 (six data values).99%, for df = 5, $t_{\alpha/2} = 4.032$

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$190 \pm 4.032 \frac{16.1}{\sqrt{6}}$$

 $\begin{array}{l} 190 \pm 26.5 \\ 163.5 < \mu < 216.5 \end{array}$

TI84 CE Plus
First, enter data on List 1
STAT, TESTS, then 8: T-Interval
then for Inpt select Data,
press Enter:
[Interval Inpt: Data Stats List:Li Freq:1 C-Level:0.99 Calculate
Interval (163.5,216.5) x̄=190 Sx=16.09968944 n=6

10. A sample of 400 racing cars showed that 80 of them cost over \$700,000. What is the 99% confidence interval for the true proportion of racing cars that cost over \$700,000?

Given x = 80, n = 400; $\hat{p} = \frac{x}{n} = \frac{80}{400} = 0.2$ $\hat{q} = 0.80$; for 99% CI the $z_{\alpha/2} = 2.576$

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$

$$0.2 \pm 2.576 \sqrt{\frac{0.2(0.8)}{400}}$$

 $0.2 \pm 0.0515 \implies 0.148$



11. A recent study of 750 internet users in Europe found that 35% of internet users were women. What is the 95% confidence interval of the true proportion of women in Europe who use the internet? **Given**: n = 750; $\hat{p} = 0.35$; $\hat{q} = 0.65$; for 95% *CI* the $z_{\alpha/2} = 1.96$

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$

$$0.35 \pm 1.96 \sqrt{\frac{0.35(0.65)}{750}}$$

$$0.35 \pm 0.0341 \implies 0.316$$

For calculators we need x; and $x = n \times \hat{p}$. Then, round x to the nearest integer. In this example: 750 $\times 0.35 = 262.5 \approx 263$



12. A random sample of 100 voters found that 46% were going to vote for a certain candidate. Find the 90% confidence interval for the population proportion of voters who will vote for that candidate. **Given**: n = 100; $\hat{p} = 0.46$ $\hat{q} = 0.54$; for 90% CI the $z_{\alpha/2} = 1.645$

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{rac{\hat{p} \cdot \hat{q}}{n}}$$

$$0.46 \pm 1.645 \sqrt{\frac{0.46(0.54)}{100}}$$

 $0.46 \pm 0.0820 \Rightarrow 0.378$

Again, for calculators we need x; and $x = n \times \hat{p}$ In this example: $100 \times 0.46 = 46$



13. In a sample of 60 mice, a biologist found that 38% were able to run a maze in 30 seconds or less. Find the 90% confidence interval for the population proportion of mice who can run a maze in 30 seconds or less. **Given**: n = 60; $\hat{p} = 0.38$ $\hat{q} = 0.62$; for 90% CI the $z_{\alpha/2} = 1.645$

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$

$$0.38 \pm 1.645 \sqrt{\frac{0.38(0.62)}{60}}$$

 $0.38 \pm 0.1031 \Rightarrow 0.277$

Again, for calculators we need x; and $x = n \times \hat{p}$ In this example: $60 \times 0.38 = 22.8 \approx 23$



14. A college believes that 28% of applicants to that school have parents who have remarried. How large a sample is needed to estimate the true proportion of students who have parents who have remarried to within 3 percentage points with 95% confidence?

$$n = \hat{p} \cdot \hat{q} \left(\frac{Z_{\alpha/2}}{E}\right)^2$$

Given $\hat{p} = 0.28$. *Error* = 3% $\Rightarrow E = 0.03$ and a 95% confidence: $z_{\alpha/2} = 1.96$

$$n = 0.28 \times 0.72 \left(\frac{1.96}{0.03}\right)^2 = 860.5 \approx 861$$

0.28*0.72(1.96/0.03)² 860.5184 15. A researcher wants to construct a 90% confidence interval for the proportion of elementary school students in Seward County who receive free or reduced-price school lunches. What sample size is needed so that the confidence interval will have a margin of error of 0.06?

$$n = \hat{p} \cdot \hat{q} \left(\frac{Z_{\alpha/2}}{E}\right)^2$$

Given: E = 0.03 and a 90% confidence: $z_{\alpha/2} = 1.645$ Since no estimation of the proportion, \hat{p} is given, use $\hat{p} = \hat{q} = 0.50$

$$n = 0.5 \times 0.5 \left(\frac{1.645}{0.06}\right)^2 = 187.9 \approx 188$$

Calculations:

0.5*0.5(1.645/0.06)² 187.9184028

- 16. The average greyhound can reach a top speed of 18.9 meters per second. A particular greyhound breeder claims her dogs are faster than the average greyhound. A sample of 50 of her dogs ran, on average, 19.5 meters per second with a population standard deviation of 1.5 meters per second. With $\alpha = 0.05$,
 - a) State the hypotheses and identify the claim.
 - b) Find the critical value.
 - c) Compute the test value.
 - d) Make the decision.
 - e) Summarize the results.

 $\begin{array}{ll} \underline{\it Info\ taken\ from\ problem}:\ \mu = 18.9 & n = 50 & \bar{x} = 19.5 & \sigma = 1.5 \\ a) \\ Ho:\ \mu = \ 18.9 \\ H_1:\ \mu \ > \ 18.9 \ \ claim \end{array}$

b) $\alpha = 0.05$ in one tail, right, $z_{\alpha} = 1.645$

c) Test Statistics: $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{19.5 - 18.9}{1.5 / \sqrt{50}} = 2.83$

d) Reject the Null Hypothesis because the test statistic falls in the critical region, that is, because |2.83| > |1.645|. Calculator: $P - Value = 0.0023 < \alpha$

e) There is sufficient evidence to support the breeder claims that her dogs are faster than the average greyhound.

On calculators, next page:



- 17. A recent report by the American Medical Association stated the average annual salary of psychiatrists is \$192,000 with a population standard deviation of \$16,000. A group of hospital administrators randomly sampled 32 psychiatrists and found an average annual salary of \$187,000. The group claims that the average annual salary is actually lower than what the American Medical Association reported. With $\alpha = 0.01$,
 - a) State the hypotheses and identify the claim.
 - b) Find the critical value.
 - c) Compute the test value.
 - d) Make the decision.
 - e) Summarize the results.

<u>Info taken from problem</u>: $\mu = 192,000$ n = 32 $\bar{x} = 187,000$ $\sigma = 16,000$ a)

 $Ho: \mu = 192,000$ $H_1: \mu < 192,000$ claim

b) $\alpha = 0.02$ in one tail, left, $z_{\alpha} = -2.326$

c) Test Statistics: $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{187000 - 192000}{16000/\sqrt{32}} = -1.77$

d) Fail to Reject Null Hypothesis because the test statistic does not fall in the critical region, that is, because |-1.77| < |-2.326|. Calculator: $P - Value = 0.039 > \alpha$

e) There is no sufficient evidence to support the administrators claim that the salary is actually lower than what the American Medical Association reported.



- 18. Sam Ying, a career counselor, claims the average number of years of schooling for an engineer is 15.8 years. A sample of 12 engineers had a mean of 15.0 years and a standard deviation of 1.5 years. Use the $\alpha = 0.05$ level of significance.
 - a) State the hypotheses and identify the claim.
 - b) Find the critical value.
 - c) Compute the test value.
 - d) Make the decision.
 - e) Summarize the results.

Given: $\mu = 15.8$ $\bar{x} = 15.0$ n = 12, s = 1.5 $\alpha = 0.05$

Answer:

 $H_0: \mu = 15.8 \ (claim).$ $H_1: \mu \neq 15.8$

b) df = 11 $\alpha = 0.05$ Critical value: $t_{\alpha/2} = \pm 2.201$

	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055

c) Test statistics:

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$t = \frac{15.0 - 15.8}{1.5/\sqrt{12}} = -1.84$$

d) Fail to reject the Null Hypothesis because the test statistic does not fall in the critical region, that is, because |-1.84| < |-2.201|. Calculator: $P - Value = 0.09 > \alpha$

e) There is no sufficient evidence to reject the counselor's claim that the average number of years of schooling for an engineer is 15.8 years.



On Calculators:

- 19. In a survey of 426 cigarette smokers, 61 of them reported that they have tried hypnosis therapy to try to quit smoking. Can you conclude that more than one-tenth of smokers have tried hypnosis therapy? Use the $\alpha = 0.05$ level of significance.
 - a) State the hypotheses and identify the claim.
 - b) Find the critical value.
 - c) Compute the test value.
 - d) Make the decision.
 - e) Summarize the results.

Given:

Significance level, $\alpha = 0.05$ in a right-tailed test $z_{\alpha} = 1.645$ $\hat{p} = 61/426 = 0.1432$ Hypotheses: $H_0: p = 0.10$ $H_1: p > 0.10$

Test Statistics:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$
$$z = \frac{0.1432 - 0.10}{\sqrt{\frac{0.10 \ (0.90)}{426}}} = 2.97$$

Notice that test statistics |2.97|>|1.645| therefore, we reject the Null. Accordingly, $p-value~=~0.001<~\alpha$

There is enough evidence to support the claim that more than one-tenth of smokers have tried hypnosis therapy.



- 20. Professor Brown teaches at U-Chem university and believes that the rate of first-time failure in his general chemistry classes is 33%. He samples 96 students from last semester who were first-time enrollees in general chemistry and finds that 23 of them failed his course. Using $\alpha = 0.05$, can you conclude that the percentage of failures differs from 33%?
 - a) State the hypotheses and identify the claim.
 - b) Find the critical value.
 - c) Compute the test value.
 - d) Make the decision.
 - e) Summarize the results.

Hypotheses:

$$H_0: p = 0.33$$
 $H_1: p \neq 0.33$

Test Statistics:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$
$$z = \frac{0.2396 - 0.33}{\sqrt{\frac{0.33 \ (0.67)}{96}}} = -1.88$$



d) Fail to reject the Null Hypothesis. Notice that test statistics |-1.88| < |-1.96| therefore, the critical value is not in the rejection region; likewise, $p - value = 0.06 > \alpha$

e) There is not enough evidence to reject the claim that the rate of first-time failure in his general chemistry classes is 33%