Hypothesis testing for a sample mean, population standard deviation σ unknown: 1-Sample T-Test.

1. Find the critical values for the following values of the significance level α, sample size n, and alternate hypothesis H_1.

   $\alpha = 0.01, \qquad n = 8, \qquad H_1: \mu \neq \mu_0$

Answer: ± 3.499
Because of $H_1: \mu \neq \mu_0$ this is a two tailed test. Using t-table check column $\alpha = 0.01$ & row for $df = 7$.

Table F The *t* Distribution

d.f.	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169

On Calculators: Inverse T. For two tailed tests we need to divide α by 2. Why? Because we need to input the area on each tail, one alpha for two tails = half of it for each tail.

CASIO 9750	*TI*84

F5 for DIST, F2 for t, and F3 for InvT
Area: $\alpha/2$
`Inverse Student-t`
`Data :Variable`
`Area :0.01÷2`
`df :7`
`Save Res:None`
`Execute`
EXE
`Inverse Student-t`
`xInv =3.4994833`
	2nd DISTR, then 4: invT
	Area: $\alpha/2$
	`invT`
	`area:0.01/2`
	`df:7`
	`Paste`
	Enter:
	`invT(0.01/2,7)`
	`-3.499483292`

2. What is the critical value for a right-tailed t test when $\alpha = 0.025$ and $n = 13$?

Answer: 2.179
This a one tailed test (right tail) with $df = 12$.

Table F The *t* Distribution

d.f.	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947

CASIO 9750

F5 for DIST, F2 for t, and F3 for InvT

```
Inverse Student-t
Data     :Variable
Area     :0.025
df       :12
Save Res:None
Execute
```

EXE

```
Inverse Student-t
xInv =2.17881283
```

Notice that the CASIO calculator output for t-critical values are always positive. If we are dealing with a right tailed test, that is what it is. For a left tail, take it as negative; for a two tailed test, plus and minus: $\pm$

*TI*84

2nd DISTR, then 4: invT

```
invT
area:0.025
df:12
Paste
```

Enter:

```
invT(0.025,12)
            -2.178812801
```

Notice that the TI Calc yields the left side, negative, t-value. Since we know that this is a right tailed test, take it as positive; otherwise, for area enter $1 - \alpha$ as follow:

```
invT
area:1-0.025
df:12
Paste
```

Enter:

```
invT(1-0.025,12)
            2.178812801
```

3. Find the critical value for the following values of the significance level α, sample size n, and alternate hypothesis H_1.

$\alpha = 0.10, \quad n = 14, \quad H_1 : \mu < \mu_0$

Answer: Left tailed test, $df = 13$ for $\alpha = 0.10$ $\quad t_\alpha = -1.350$

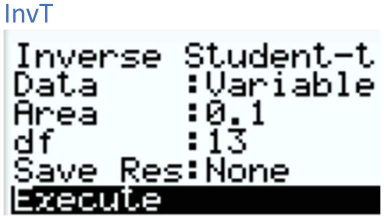
Note: Why negative? Table only shows absolute values. For a right tailed test, they are positive; for a left tailed test, negative; for a two-tailed test, both, positive and negative.

Table F The *t* Distribution

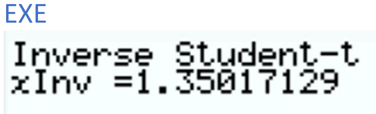
d.f.	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977

CASIO 9750

F5 for DIST, F2 for t, and F3 for InvT

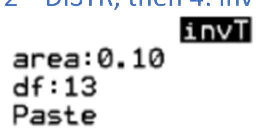


EXE

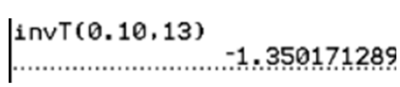


*TI*84

2nd DISTR, then 4: invT



Enter:



4. Reginald Brown, an inspector from the Department of Weights and Measures, weighed 15 eighteen-ounce cereal boxes of corn flakes. He found their mean weight to be 17.8 ounces with a standard deviation of 0.4 ounces. At $\alpha = 0.01$, are the cereal boxes lighter than they should be?

Given: $\mu = 18 \quad \bar{x} = 17.8 \quad n = 15, \quad s = 0.4 \quad \alpha = 0.01$

Answer:

$H_0: \mu = 18.0$ (claim)

$H_1: \mu < 18.0$

$df = 14 \quad \alpha = 0.01$ Critical value: $t_\alpha = -2.624$

	Confidence intervals	80%	90%	95%	98%
	One tail, α	0.10	0.05	0.025	0.01
d.f.	Two tails, α	0.20	0.10	0.05	0.02
1		3.078	6.314	12.706	31.821
2		1.886	2.920	4.303	6.965
3		1.638	2.353	3.182	4.541
4		1.533	2.132	2.776	3.747
5		1.476	2.015	2.571	3.365
6		1.440	1.943	2.447	3.143
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9		1.383	1.833	2.262	2.821
10		1.372	1.812	2.228	2.764
11		1.363	1.796	2.201	2.718
12		1.356	1.782	2.179	2.681
13		1.350	1.771	2.160	2.650
14		1.345	1.761	2.145	2.624
15		1.341	1.753	2.131	2.602

Test value:

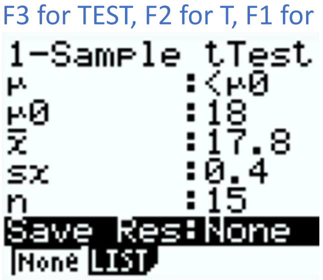
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{17.8 - 18}{0.4/\sqrt{15}} = -1.94$$

On Calculators:

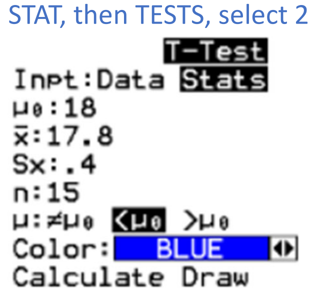
CASIO 9750

F3 for TEST, F2 for T, F1 for 1-S



TI84

STAT, then TESTS, select 2: T-TEST



EXE	Enter:
1-Sample tTest μ <18 t =-1.9364917 p =0.03662922 x̄ =17.8 sx =0.4 n =15	T-Test μ<18 t=-1.936491673 p=.0366292195 x̄=17.8 Sx=.4 n=15

Do not reject the claim since -1.94 does not fall within the critical region; that is $|-1.94| < |-2.64|$
Also notice that p-value $= \ 0.036 \ > \ \alpha = \ 0.01$
Therefore, there is not enough evidence to reject the claim that the cereal boxes weigh 18 ounces.

5. Science fiction novels average 290 pages in length. The average length of 14 randomly chosen novels written by I. M. Wordy was 305 pages in length with a standard deviation of 35. At $\alpha = \ 0.05$, are Wordy's novels significantly longer than the average science fiction novel?

Given: $\mu = 290 \quad \bar{x} = 305 \quad n = 14, \quad s = 35 \quad \alpha = \ 0.05$

Answer:

$H_0: \mu = 290$
$H_1: \mu > 290 \ \ (claim)$

$df \ = 13 \ \ \alpha = \ 0.05$ Critical value: $t_\alpha = 1.771$,

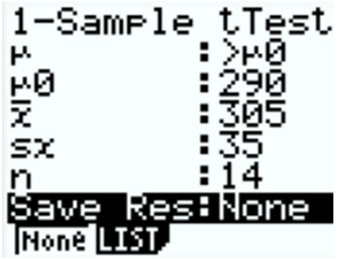
	One tail, α	0.10	0.05	0.025	0.01
d.f.	Two tails, α	0.20	0.10	0.05	0.02
1		3.078	6.314	12.706	31.821
2		1.886	2.920	4.303	6.965
3		1.638	2.353	3.182	4.541
4		1.533	2.132	2.776	3.747
5		1.476	2.015	2.571	3.365
6		1.440	1.943	2.447	3.143
7		1.415	1.895	2.365	2.998
8		1.397	1.860	2.306	2.896
9		1.383	1.833	2.262	2.821
10		1.372	1.812	2.228	2.764
11		1.363	1.796	2.201	2.718
12		1.356	1.782	2.179	2.681
13		1.350	1.771	2.160	2.650
14		1.345	1.761	2.145	2.624

Test value:

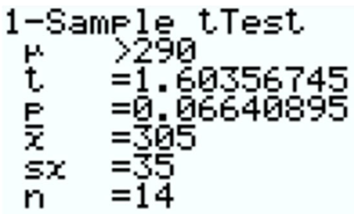
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{305 - 290}{35/\sqrt{14}} = 1.60$$

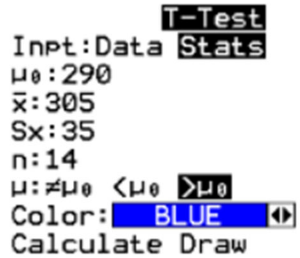
CASIO 9750
F3 for TEST, F2 for T, F1 for 1-S



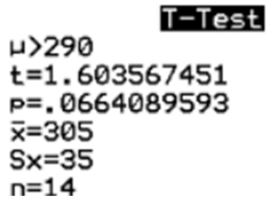
EXE



TI84
STAT, then TESTS, select 2: T-TEST



Enter:



The conclusion is to not reject the null hypothesis because 1.60 does not fall within the critical region; that is $|1.60| < |1.771|$

By the calculator, p-value $= 0.066 > \alpha = 0.05$

There is not enough evidence to support the claim that Wardy's novels are longer than the average science fiction novel.

6. The mean annual tuition and fees for a sample of 8 private colleges was \$31,900 with a standard deviation of \$5500. A dotplot shows that it is reasonable to assume that the population is approximately normal. You wish to test whether the mean tuition and fees for private colleges is different from \$35,300.

   i) State the null and alternate hypotheses.
   ii) Compute the value of the test statistic and state the number of degrees of freedom.
   iii) State a conclusion regarding H_0. Use the $\alpha = 0.05$ level of significance.

Given: $\mu = 35{,}300$ $\quad \bar{x} = 31{,}900$ $\quad n = 8,$ $\quad s = 5{,}500$ $\quad \alpha = 0.05$

$H_0: \mu = 35300$
$H_1: \mu \neq 35300 \ (claim)$

Test value:

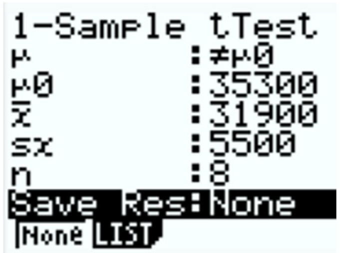
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{31900 - 35300}{5500/\sqrt{8}} = -1.748$$

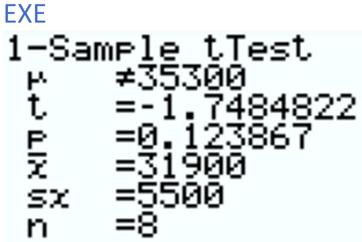
$Df = n - 1 = 7$
$\alpha = 0.05$ Critical value: $t_{\alpha/2} = \pm 2.365$

d.f.	One tail, α	0.10	0.05	0.025
	Two tails, α	0.20	0.10	0.05
1		3.078	6.314	12.706
2		1.886	2.920	4.303
3		1.638	2.353	3.182
4		1.533	2.132	2.776
5		1.476	2.015	2.571
6		1.440	1.943	2.447
7		1.415	1.895	2.365
8		1.397	1.860	2.306

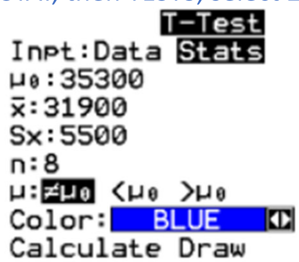
CASIO 9750
F3 for TEST, F2 for T, F1 for 1-S



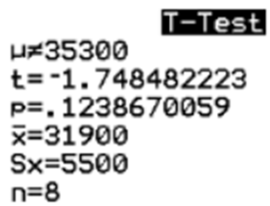
EXE



TI84
STAT, then TESTS, select 2: T-TEST



Enter:



Do not reject the claim since -1.748 does not fall within the critical region; that is $|-1.748| < |-2.365|$
Also notice that p-value $= 0.1238 > \alpha = 0.05$
Therefore, there is insufficient evidence to conclude that the mean annual tuition and fees is different from \$35,300.

7. Historically, a certain region has experienced 97 thunder days annually. (A "thunder day" is day on which at least one instance of thunder is audible to a normal human ear). Over the past six years, the mean number of thunder days is 73 with a standard deviation of 29. Can you conclude that the mean number of thunder days is less than 97? Use the $\alpha = 0.10$ level of significance.
A) No. There is insufficient evidence to conclude that the number of thunder days is less than 97.
B) Yes. The number of thunder days appears to be less than 97.
C) There is not enough information to draw a conclusion.

Given: $\mu = 97$ $\quad \bar{x} = 73$ $\quad n = 6,$ $\quad s = 29$ $\quad \alpha = 0.10$

$H_0: \mu = 97$
$H_1: \mu < 97$

Test value:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{73 - 97}{29/\sqrt{6}} = -2.03$$

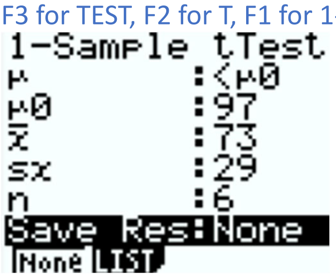
$Df = n - 1 = 5$

$\alpha = 0.10$ Critical value: $t_\alpha = -1.476$

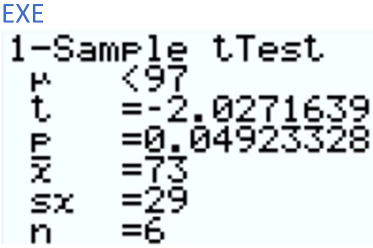
d.f.	One tail, α	0.10	0.05	0.025
	Two tails, α	0.20	0.10	0.05
1		3.078	6.314	12.706
2		1.886	2.920	4.303
3		1.638	2.353	3.182
4		1.533	2.132	2.776
5		1.476	2.015	2.571
6		1.440	1.943	2.447
7		1.415	1.895	2.365
8		1.397	1.860	2.306

CASIO 9750

F3 for TEST, F2 for T, F1 for 1-S

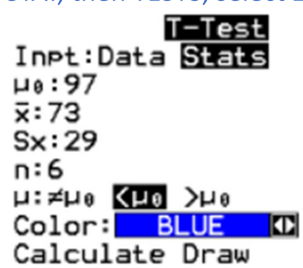


EXE

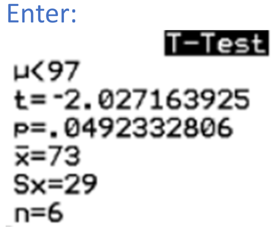


TI84

STAT, then TESTS, select 2: T-TEST



Enter:



Reject the Null Hypothesis because -2.03 falls within the critical region; that is $|-2.03| > |-1.476|$

And $p - value = 0.049 < \alpha = 0.10$

B) Yes, reject the null hypothesis: the number of thunder days appears to be less than 97.

8. A machine that fills beverage cans is supposed to put 24 ounces of beverage in each can. Following are the amounts measured in a simple random sample of eight cans.

   24.00 23.94 23.96 23.98 23.91 23.90 23.83 23.95

   Assume that the sample is approximately normal. Can you conclude that the mean volume differs from 24 ounces? Use the $\alpha = 0.1$ level of significance.

   A) No. There is insufficient evidence to conclude that the mean fill volume differs from24 ounces.
   B) There is not enough information to draw a conclusion.
   C) Yes. The mean fill volume appears to differ from 24 ounces.

……

Unless you are using a Graphing Calculator, you need to find the sample mean, $\bar{x}$, and the sample standard deviation, s. The number of observations is $n = 8$.

Results: $\bar{x} = 23.934$

$s = 0.053$

$H_0: \mu = 24$

$H_1: \mu \neq 24 \; (claim)$

Test value:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

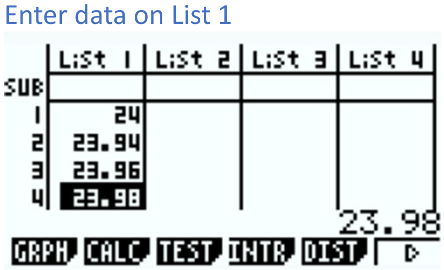
$$t = \frac{23.934 - 24}{0.053/\sqrt{8}} = -3.51$$

$Df = n - 1 = 7$; $\alpha = 0.1$, two-tailed test. T-critical value: $t_{\alpha/2} = \pm 1.895$

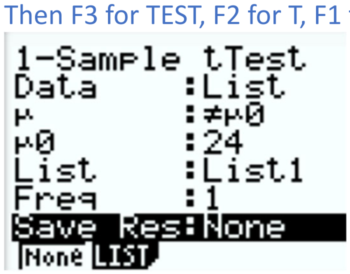
d.f.	One tail, α	0.10	0.05	0.025
	Two tails, α	0.20	0.10	0.05
1		3.078	6.314	12.706
2		1.886	2.920	4.303
3		1.638	2.353	3.182
4		1.533	2.132	2.776
5		1.476	2.015	2.571
6		1.440	1.943	2.447
7		1.415	1.895	2.365
8		1.397	1.860	2.306

CASIO 9750

Enter data on List 1

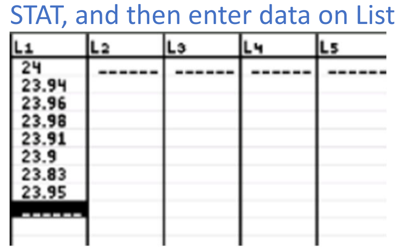


Then F3 for TEST, F2 for T, F1 for 1-S and F1 for LIST



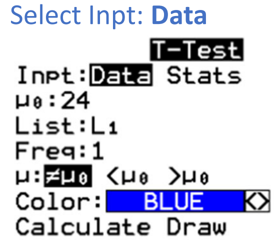
TI84

STAT, and then enter data on List 1



STAT, then TESTS, select 2: T-TEST

Select Inpt: **Data**



EXE	Enter:
1-Sample tTest μ ≠24 t =-3.5067165 p =9.9031E-03 x̄ =23.93375 sx =0.05343554 n =8	T-Test μ≠24 t=-3.506716507 p=0.0099030549 x̄=23.93375 Sx=0.0534355419 n=8

Reject the Null Hypothesis because -3.51 falls within the critical region; that is $|-3.51| > |-1.895|$
And $p - value \ = \ 0.0099 \ \approx 0.01 < \ \alpha \ = 0.10$
C) Yes. The mean fill volume appears to differ from 24 ounces.