

*Hypothesis testing for a sample mean, population standard deviation  $\sigma$  known: 1-Sample Z-Test.*

$$\text{Test Statistics: } Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

1. A recent survey of gasoline prices indicated that the national average was \$4.098 per gallon. The Dallas Automobile Club claimed that gasoline in Texas was significantly lower than the national average. A survey covering 10 different suburbs in Dallas found the average price of gasoline to be \$3.924 per gallon with a population standard deviation of \$0.33, At  $\alpha = 0.05$ :
  - a) State the hypotheses and identify the claim.
  - b) Find the critical value.
  - c) Compute the test value.
  - d) Make the decision.
  - e) Summarize the results.

*Info taken from problem:*  $\mu = 4.098$     $n = 10$     $\bar{x} = 3.924$     $\sigma = 0.33$

a)

$$H_0: \mu = 4.098$$

$$H_1: \mu < 4.098 \text{ claim}$$

b)  $\alpha = 0.05$  in one tail, left,  $z_\alpha = -1.645$

$$\text{c) Test Statistics: } z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{3.924 - 4.098}{0.33 / \sqrt{10}} = -1.67$$

d) Reject the Null Hypothesis because the test statistic falls in the critical region, that is, because  $|-1.67| > |-1.645|$ , we can reject the null hypothesis in favor of the alternative.

OR we can say based on the p-value output of the calculator that we reject the Null Hypothesis because the test p-value (0.048) is less than the significance level ( $\alpha = 0.05$ ).

e) There is sufficient evidence to support the claim that gasoline in Texas was significantly lower than the national average.

***On Calculators (next page):***

<p><i>CASIO 9750:</i> F3 for TEST, F1 for Z then F1 for I-S:</p> <pre>1-Sample ZTest Data :Variable μ      :&lt;μ₀ μ₀     :4.098 σ      :0.33 x̄      :3.924 n      :10 List Var</pre> <p><i>EXE:</i></p> <pre>1-Sample ZTest μ      &lt;4.098 z      =-1.6673828 P      =0.04771915 x̄      =3.924 n      =10</pre>	<p><i>TI84</i> STAT, then TESTS, then 1: Z-TEST, select Stats:</p> <pre>Z-Test Inpt:Data Stats μ₀:4.098 σ:.33 x̄:3.924 n:10 μ:≠μ₀ &lt;μ₀ &gt;μ₀ Color: BLUE Calculate Draw</pre> <p><i>Enter:</i></p> <pre>Z-Test μ&lt;4.098 z=-1.667382766 P=.0477191371 x̄=3.924 n=10</pre>
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2. A recent survey indicated that the average amount spent for breakfast by business managers was \$7.58 with a standard deviation of \$0.42. It was felt that breakfasts on the West Coast were higher than \$7.58. A sample of 81 business managers on the West Coast had an average breakfast cost of \$7.65. At  $\alpha = 0.05$ :
- State the hypotheses and identify the claim.
  - Find the critical value.
  - Compute the test value.
  - Make the decision.
  - Summarize the results.

Info taken from problem:  $\mu = 7.58$   $n = 81$   $\bar{x} = 7.65$   $\sigma = 0.42$

a)

$$H_0: \mu = 7.58$$

$$H_1: \mu > 7.58 \text{ claim}$$

b)  $\alpha = 0.05$  in one tail, right,  $z_{\alpha} = 1.645$

c) Test Statistics:  $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{7.65 - 7.58}{0.42 / \sqrt{81}} = 1.50$

d) Fail to reject the Null Hypothesis because the test statistic does not falls in the critical region, that is, because  $|1.50| < |1.645|$ . OR we can say based on the p-value output of the calculator that fail to reject the Null Hypothesis because the test p-value (0.066) is greater than the significance level ( $\alpha = 0.05$ ).

e) There is no sufficient evidence to support the claim that breakfasts on the West Coast were higher than \$7.58.

**On Calculators (next page):**

<p><i>CASIO 9750:</i>  <i>F3 for TEST, F1 for Z</i>  <i>then F1 for 1-S:</i></p> <pre> 1-Sample ZTest μ₀      : &gt;μ₀ σ      : 7.58 σ      : 0.42 x̄      : 7.65 n      : 81 Save Res: None None LIST </pre> <p><i>EXE:</i></p> <pre> 1-Sample ZTest μ      : &gt;7.58 z      : 1.5 P      : 0.0668072 x̄      : 7.65 n      : 81 </pre>	<p><i>TI84</i>  <i>STAT, then TESTS,</i>  <i>then 1: Z-TEST, select</i>  <i>Stats:</i></p> <pre> Z-Test Inpt: Data Stats μ₀: 7.58 σ: .42 x̄: 7.65 n: 81 μ: ≠μ₀ &lt;μ₀ &gt;μ₀ Color: BLUE Calculate Draw </pre> <p><i>Enter:</i></p> <pre> Z-Test μ&gt;7.58 z=1.5 P=.0668072287 x̄=7.65 n=81 </pre>
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3. At a water bottling facility, a technician is testing a bottle filling machine that is supposed to deliver 1000 milliliters of water. The technician dispenses 20 samples of water and determines the volume of each sample. The 20 samples have a mean volume of  $\bar{x} = 1002.1$  mL. The machine is out of calibration if the mean volume differs from 1000 mL. The technician wants to perform a hypothesis test to determine whether the machine is out of calibration. The standard deviation of the dispensed volume is known to be  $\sigma = 3.5$ . At  $\alpha = 0.02$ :
- State the hypotheses and identify the claim.
  - Compute the test value.
  - Find the P-value.
  - Make the decision.
  - Summarize the results.

Info taken from problem:  $\mu = 1000$     $n = 20$     $\bar{x} = 1002.1$     $\sigma = 3.5$

a)

$H_0: \mu = 1000$

$H_1: \mu \neq 1000$  *claim*

b)  $\alpha = 0.02$  in two tails,  $z_{\alpha/2} = \pm 2.326$

c) Test Statistics:  $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{1002.1 - 1000}{3.5 / \sqrt{20}} = 2.68$

d) Reject the Null Hypothesis because the test statistic falls in the critical region, that is, because  $|2.68| > |2.326|$ , we can reject the null hypothesis in favor of the alternative.

OR we can say based on the p-value output of the calculator that we reject the Null Hypothesis because the test p-value (0.007) is less than the significance level ( $\alpha = 0.02$ ).

e) There is sufficient evidence to support the claim that the machine is out of calibration.

**On Calculators (next page):**

<p><i>CASIO 9750:</i>  <i>F3 for TEST, F1 for Z</i>  <i>then F1 for 1-S:</i></p> <pre> 1-Sample ZTest μ      :≠μ₀ μ₀     :1000 σ      :3.5 x̄      :1002.1 n      :20 Save Res:None None LIST </pre> <p><i>EXE:</i></p> <pre> 1-Sample ZTest μ      ≠1000 z      =2.68328157 P      =7.2904E-03 x̄      =1002.1 n      =20 </pre>	<p><i>TI84</i>  <i>STAT, then TESTS,</i>  <i>then 1: Z-TEST, select</i>  <i>Stats:</i></p> <pre> Z-Test Inpt:Data Stats μ₀:1000 σ:3.5 x̄:1002.1 n:20 μ:≠μ₀ &lt;μ₀ &gt;μ₀ Color: BLUE Calculate Draw </pre> <p><i>Enter:</i></p> <pre> Z-Test μ≠1000 z=2.683281573 P=.007290452 x̄=1002.1 n=20 </pre>
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4. The Eagle Ridge Contractors Association claims the average price of a home in their subdivision is \$525,000 with a standard deviation of \$12,500. A sample of 10 homes for sale in this subdivision had an average selling price of \$510,550. The Eagle Ridge Home Owners Association is interested in knowing if the costs of homes for sale in this subdivision are actually lower than claimed? At  $\alpha = 0.05$ :
- State the hypotheses and identify the claim.
  - Compute the test value.
  - Find the P-value.
  - Make the decision.
  - Summarize the results.

Info taken from problem:  $\mu = 525,000$     $n = 10$     $\bar{x} = 510,550$     $\sigma = 12,500$

a)

$H_0: \mu = 525,000$

$H_1: \mu < 525,000$  *claim*

b)  $\alpha = 0.05$  in one tail, left:  $z_\alpha = -1.645$

c) Test Statistics:  $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{510550 - 525000}{12500 / \sqrt{10}} = -3.66$

d) Reject the Null Hypothesis because the test statistic falls in the critical region, that is, because  $|-3.66| > |-1.645|$ , we can reject the null hypothesis in favor of the alternative.

OR we can say based on the p-value output of the calculator that we reject the Null Hypothesis because the test p-value (0.0001) is less than the significance level ( $\alpha = 0.05$ ).

e) There is sufficient evidence to support the claim of the Home Owners Association that the costs of homes for sale in this subdivision are actually lower.

***On Calculators (next page):***

<p><i>CASIO 9750:</i> F3 for TEST, F1 for Z then F1 for 1-S:</p> <pre> 1-Sample ZTest μ₀      : &lt;μ₀ σ      : 125000 x̄      : 510550 n      : 10 Save Res: None None LIST </pre> <p><i>EXE:</i></p> <pre> 1-Sample ZTest μ      &lt;525000 z      =-3.655593 P      =1.2829E-04 x̄      =510550 n      =10 </pre>	<p><i>TI84</i> STAT, then TESTS, then 1: Z-TEST, select Stats:</p> <pre> Z-Test Inpt:Data Stats μ₀:525000 σ:12500 x̄:510550 n:10 μ:≠μ₀ &lt;μ₀ &gt;μ₀ Color: BLUE Calculate Draw </pre> <p><i>Enter:</i></p> <pre> Z-Test μ&lt;525000 z=-3.655592975 P=1.2832741E-4 x̄=510550 n=10 </pre>
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5. At a certain university, the average cost of books was \$370 per student last semester and the population standard deviation was \$90. This semester a sample of 40 students revealed an average cost of books of \$400 per student. The Dean of Students believes that the costs are greater this semester. At alpha = 0.01:
- State the hypotheses and identify the claim.
  - Compute the test value.
  - Find the P-value.
  - Make the decision.
  - Summarize the results.

Info taken from problem:  $\mu = 370$   $n = 40$   $\bar{x} = 400$   $\sigma = 90$

a)

$H_0: \mu = 370$

$H_1: \mu > 370$  *claim*

b)  $\alpha = 0.01$  in one tail, right:  $z_\alpha = 2.326$

c) Test Statistics:  $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{400 - 370}{90 / \sqrt{40}} = 2.11$

d) Fail to reject the Null Hypothesis since test statistics absolute value  $2.11 < \text{critical value } 2.326$

Fail to reject the Null Hypothesis because the test statistic does not falls in the critical region, that is, because  $|2.11| < |2.326|$ . OR we can say based on the p-value output of the calculator that fail to reject the Null Hypothesis because the test p-value (0.0175) is greater than the significance level ( $\alpha = 0.01$ ).

e) There is no sufficient evidence to support the Dean's claim that the costs of books are greater this semester

***On Calculators (next page):***

CASIO 9750:  
F3 for TEST, F1 for Z  
then F1 for 1-S:

```
1-Sample ZTest
μ₀: >μ₀
μ₀: 370
σ: 90
x̄: 400
n: 40
Save Res: None
[None] [LIST]
```

EXE:

```
1-Sample ZTest
μ > μ₀
Z = 2.10818511
P = 0.01750749
x̄ = 400
n = 40
```

TI84  
STAT, then TESTS, then  
1: Z-TEST, select  
Stats:

```
Z-Test
Inpt: Data Stats
μ₀: 370
σ: 90
x̄: 400
n: 40
μ: ≠μ₀ <μ₀ >μ₀
Color: BLUE
Calculate Draw
```

Enter:

```
Z-Test
μ > 370
z = 2.108185107
P = 0.0175074283
x̄ = 400
n = 40
```