1) The average gas mileage of a certain model car is 30.0 miles per gallon. If the gas mileages are normally distributed with a standard deviation of 0.75 miles per gallon, find the <u>probability that a car</u> has a gas mileage of between 29.8 and 30.2 miles per gallon.

Finding both z scores, for $x = 29.8$ and 30.2:

$z = \frac{x-\mu}{\sigma} = \frac{30.2-30.0}{0.75} = 0.26666\ldots \approx 0.27$ Therefore, $P(x < 30.2) \;=>\; P(z < 0.27) = 0.6064$

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

$$z = \frac{x-\mu}{\sigma} = \frac{29.8-30.0}{0.75} = -0.26666\ldots \approx -0.27$$

$P(x < 29.8) \;=>\; P(z < -0.27) = 0.3936$

−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

$P(29.8 < x < 30.2) \Rightarrow P(-0.27 < z < 0.27) = 0.6064 - 0.3936 = 0.2128 \approx 0.213$ to three decimal places.

On Calculators:

Normal C.D Data :Variable Lower :29.8 Upper :30.2 σ :0.75 μ :30 Save Res:None None LIST $CASIO$ 9750	Normal C.D p =0.21027417 z:Low=-0.2666666 z:Up =0.26666666	normalcdf lower:29.8 upper:30.2 μ:30 σ:0.75 Paste $TI84$	normalcdf(29.8,30.2,30,0.▸ .210274064

2) A certain car model has a mean gas mileage of 29 miles per gallon (mpg) with a standard deviation 3 mpg. A pizza delivery company buys 49 of these cars. What is the probability that the average mileage of the fleet is greater than 28.8 mpg?

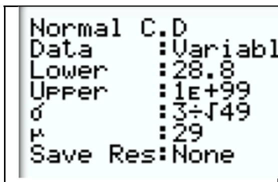
Sample size $n = 49$, $\bar{x} = 28.8$ $\mu = 29$ $\sigma = 3$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{28.8 - 29}{3/\sqrt{49}} = -0.46666 \ldots \approx= -0.47$$

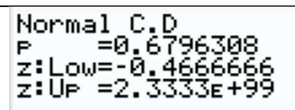
$$P(\bar{x} > 28.8) = 1 - P(\bar{x} < 28.8) = 1 - P(x < -0.47) = 1 - 0.3192 = 0.6808$$

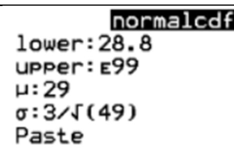
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859

On Calculators:

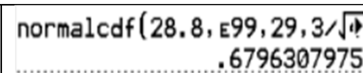


CASIO 9750





*TI*84



3) A ferry will safely accommodate 68 tons of passenger cars. Assume that the mean weight of a passenger car is 1.8 tons with standard deviation 0.5 tons. If a random sample of 35 cars are loaded onto the ferry, what is the probability that the maximum safe weight will be exceeded?

….

The probability that the 35 cars weigh more than 68 tons is the same as asking for the probability that their average weight is greater than 68/35 = 1.943 tons. That is, if the 35 cars weight 1.943 ton as average, we are ok since that the max allowed for the ferry. But the average weight is 1.8 ton for all car. It seems that we are safe (not exceed the max weight); however, 1.8 ton is the average, cars may weight much more than that. What is the probability of exceeding the average?

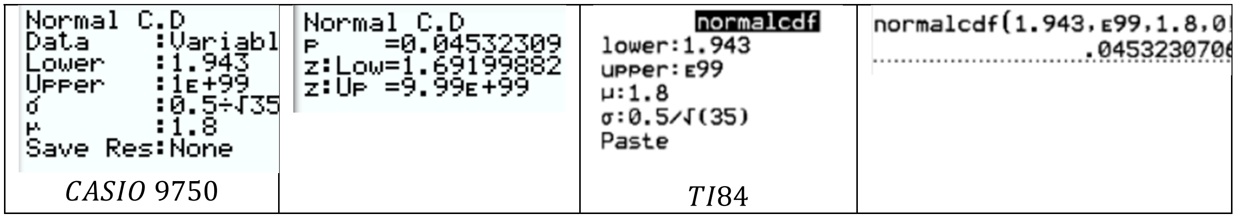
Sample size $n = 35$, $\bar{x} = 1.943$ $\mu = 1.8$ $\sigma = 0.5$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1.943 - 1.8}{0.5/\sqrt{35}} = 1.69$$

$$P(\bar{x} > 1.94) = 1 - P(\bar{x} < 1.94) = 1 - P(\bar{x} < 1.69) = 1 - 0.9545 = 0.0455$$

1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545

On Calculators:



The probability that the maximum safe weight will be exceeded is approximately 0.0455 or 4.55%.

4) A <u>sample of size 80</u> will be drawn from a population with mean 23 and standard deviation 13. Find the probability that $\bar{x}$ will be between 22 and 25.

Sample size $n = 80$, $\mu = 23$ $\sigma = 13$ $P_{22<\bar{x}<25}$?

Z score for $\bar{x} = 25$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{25 - 23}{13/\sqrt{80}} = 1.37$$

Z score for $\bar{x} = 22$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{22 - 23}{13/\sqrt{80}} = -0.688 \ldots \approx -0.69$$

$$P_{22<\bar{x}<25} = P_{x<25} - P_{x<22} = 0.9147 - 0.2451 = 0.6696$$

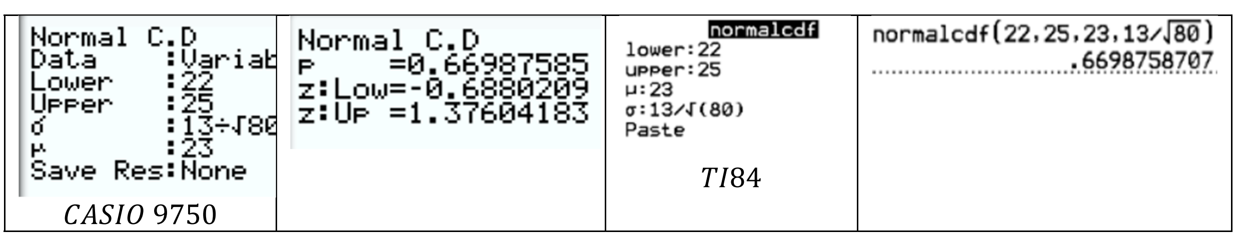
$P(z < 1.37)$

1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

$P(z < -0.69)$

−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776

On Calculators:



5) <u>A sample of size 48</u> will be drawn from a population with mean 20 and standard deviation 5. Find the probability that $\bar{x}$ will be greater than 21.

Sample size $n = 48, \quad \bar{x} = 21 \qquad \mu = 20 \qquad \sigma = 5$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{21-20}{5/\sqrt{48}} = 1.39$$

$$P_{(\bar{x}>21)} = 1 - P(z < 1.39) = 1 - 0.9177 = 0.0823$$

1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

On Calculators:

Normal C.D Data :Variabl Lower :21 Upper :1E+99 σ :5÷√48 μ :20 Save Res:None *CASIO* 9750	Normal C.D p =0.08292833 z:Low=1.38564065 z:Up =1.3856E+99	normalcdf lower:21 upper:E99 μ:20 σ:5/√(48) Paste *TI*84	normalcdf(21,E99,20,5/√48 .082928384

6) <u>A sample of size 47</u> will be drawn from a population with mean 25 and standard deviation 5. Find the probability that $\bar{x}$ will be less than 26.

Sample size $n = 47, \quad \bar{x} = 26 \qquad \mu = 25 \qquad \sigma = 5$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{26-25}{5/\sqrt{47}} = 1.37$$

$$P_{(\bar{x}<26)} = P(z < 1.37) = 0.9147$$

1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

On Calculators:

Normal C.D Data :Variable Lower :-1E+99 Upper :26 σ :5÷√47 μ :25 Save Res:None *CASIO* 9750	Normal C.D p =0.91483292 z:Low=-1.371E+99 z:Up =1.37113092	normalcdf lower:-E99 upper:26 μ:25 σ:5/√(47) Paste *TI*84	normalcdf(-E99,26,25,5/√47 .914832869

7) The mean annual income for people in a certain city (in thousands of dollars) is 38, with a standard deviation of 33. A pollster draws a sample of 39 people to interview. What is the probability that the sample mean income is between 36 and 40 (thousands of dollars)?

Sample size $n = 39$, $\mu = 38$ $\sigma = 33$ $P_{36<\bar{x}<40}$?

Z score for $\bar{x} = 36$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{36-38}{33/\sqrt{39}} = 0.3784\ldots \approx 0.38$$

Z score for $\bar{x} = 40$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{40-38}{33/\sqrt{39}} = -\,0.3784\ldots \approx -0.38$$

$$P_{36<\bar{x}<40} = P_{x<40} - P_{x<36} = 0.6480 - 0.3520 = 0.2960$$

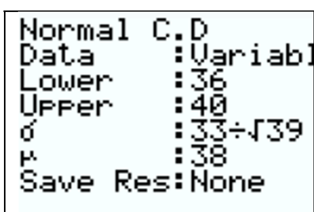
$P(z < 0.38)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517

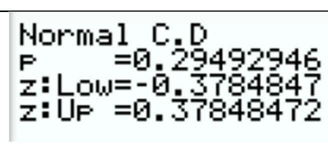
$P(z < -0.38)$

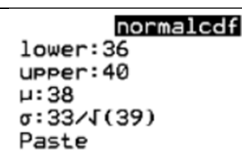
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859

On Calculators:



CASIO 9750





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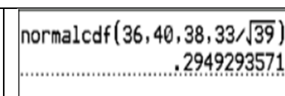


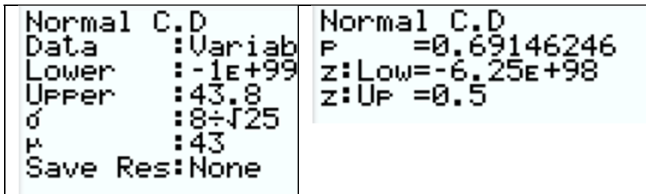
8) The average age of doctors in a certain hospital is 43.0 years old. Suppose the distribution of ages is normal and has a standard deviation of 8.0 years. If <u>25 doctors</u> are chosen at random for a committee, find the probability that the average age of those doctors is less than 43.8 years. Assume that the variable is normally distributed.

Sample size $n = 25$, $\bar{x} = 43.8$ $\mu = 43$ $\sigma = 8$

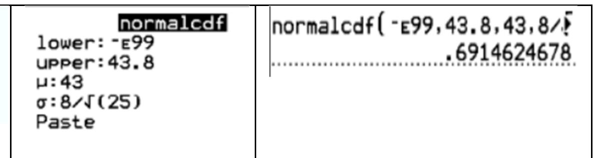
$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{43.8-43}{8/\sqrt{25}} = 0.5$$

$P_{(\bar{x}<43.8)} = P_{(z<0.5)} = 0.6915$ As percentage: $0.6915 \times 100 = 69.15\%$ or 69.2%

0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549



CASIO 9750



*TI*84