

- 1) A normal population has a mean  $\mu = 31$  and standard deviation  $\sigma = 8$ . What proportion of the population is less than 29?

In order to use tables, we need to find the z score:

$$z = \frac{x - \mu}{\sigma} = \frac{29 - 31}{8} = -0.25$$

Therefore,  $P(x < 29) = P(z < -0.25)$ . By the table:

-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Answer: 0.4013

On Calculators:

<pre>Normal C.D Data :Variable Lower :-1E+99 Upper :29 σ :8 μ :31 Save Res:None None LIST CASIO 9750</pre>	<pre>Normal C.D P =0.40129367 z:Low=-1.25E+98 z:Up =-0.25</pre>	<pre>normalcdf lower:-1E99 upper:29 μ:31 σ:8 Paste T184</pre>	<pre>normalcdf(-1E99,29,31,8) .....4012937256</pre>
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- 2) A normal population has a mean  $\mu = 33$  and standard deviation  $\sigma = 8$ . What is the probability that a randomly chosen value will be greater than 30?

Finding the z score:

$$z = \frac{x - \mu}{\sigma} = \frac{30 - 33}{8} = -0.375 \approx -0.38 \quad \text{Therefore, } P(x < 30) \Rightarrow P(z < -0.38):$$

-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859

$$P(z < -0.38) = 0.3520$$

And

$$P(z > -0.38) = 1 - 0.3520 = 0.6480$$

Answer: 0.6480

<pre>Normal C.D Data :Variable Lower :30 Upper :1E+99 σ :8 μ :33 Save Res:None None LIST CASIO 9750</pre>	<pre>Normal C.D P =0.64616976 z:Low=-0.375 z:Up =1.25E+98</pre>	<pre>normalcdf lower:30 upper:E99 μ:33 σ:8 Paste T184</pre>	<pre>normalcdf(30,E99,33,8) .....6461697127</pre>
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Notice that the output on the calculators correct to four decimal places is 0.6462

- 3) The average length of crocodiles in a swamp is 12.5 feet. If the lengths are normally distributed with a standard deviation of 2.1 feet, find the probability that a crocodile is more than 12 feet long.

$$z = \frac{x - \mu}{\sigma} = \frac{12 - 12.5}{2.1} = -0.238 \approx -0.24$$

On the table,  $P(z < -0.24)$ :

-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247

Therefore,  $P(z > -0.24) = 1 - 0.4052 = 0.5948$

**On Calculators:**

<pre>Normal C.D Data  :Variable Lower :12 Upper :1E+99 σ      :2.1 μ      :12.5 Save Res:None None LIST CASIO 9750</pre>	<pre>Normal C.D P      =0.59409638 z:Low=-0.2380952 z:UP  =4.7619E+98</pre>	<pre>normalcdf lower:12 upper:1E99 μ:12.5 σ:2.1 Paste T184</pre>	<pre>normalcdf(12, 1E99, 12.5, 2.1) ..... .5940963403</pre>
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- 4) The average gas mileage of a certain model car is 30.0 miles per gallon. If the gas mileages are normally distributed with a standard deviation of 0.75 miles per gallon, find the probability that a car has a gas mileage of between 29.8 and 30.2 miles per gallon.

Finding both z scores, for  $x = 29.8$  and  $30.2$ :

$$z = \frac{x - \mu}{\sigma} = \frac{30.2 - 30.0}{0.75} = 0.26666 \dots \approx 0.27 \quad \text{Therefore, } P(x < 30.2) \Rightarrow P(z < 0.27) = 0.6064$$

TABLE A-2 (continued) Cumulative Area from the LEFT										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

$$z = \frac{x - \mu}{\sigma} = \frac{29.8 - 30.0}{0.75} = -0.26666 \dots \approx -0.27$$

$P(x < 29.8) \Rightarrow P(z < -0.27) = 0.3936$

-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

$P(29.8 < x < 30.2) \Rightarrow P(-0.27 < z < 0.27) = 0.6064 - 0.3936 = 0.2128 \approx 0.213$  to three decimal places.

**On Calculators:**

<pre>Normal C.D Data :Variable Lower :29.8 Upper :30.2 σ :0.75 μ :30 Save Res:None None LIST CASIO 9750</pre>	<pre>Normal C.D P =0.21027417 z:Low=-0.26666666 z:Up =0.26666666</pre>	<pre>normalcdf lower:29.8 upper:30.2 μ:30 σ:0.75 Paste T184</pre>	<pre>normalcdf(29.8,30.2,30.0, .....) .210274064</pre>
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- 5) The average height of flowering cherry trees in a certain nursery is 9.5 feet. If the heights are normally distributed with a standard deviation of 1.3 feet, find the probability that a tree is less than 11.5 feet tall.

$$z = \frac{x - \mu}{\sigma} = \frac{11.5 - 9.5}{1.3} = 1.53846 \dots \approx 1.54$$

$P(x < 11.5) \Rightarrow P(z < 1.54)$ , by table:

1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

**Answer: 0.9382**

**On Calculators:**

<pre>Normal C.D Data :Variable Lower :-1E+99 Upper :11.5 σ :1.3 μ :9.5 Save Res:None None LIST CASIO 9750</pre>	<pre>Normal C.D P =0.93803209 z:Low=-7.692E+98 z:Up =1.53846154</pre>	<pre>normalcdf lower:-E99 upper:11.5 μ:9.5 σ:1.3 Paste T184</pre>	<pre>normalcdf(-E99,11.5,9.5,1) .....) .9380320809</pre>
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- 6) For the standard normal curve, find the z-score that corresponds to the 90th percentile.

Look up on the table for the z score that corresponds to 90% or 0.9000

0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

0.8997 is the closest number to 0.9000. The corresponding z score is 1.28

On calculators, use Inverse Normal. For the standard normal distribution, use  $\mu = 0$  and  $\sigma = 1$ .

<pre>Inverse Normal Data :Variable Tail :Left Area :0.9 σ :1 μ :0 Save Res:None None LIST CASIO 9750</pre>	<pre>Inverse Normal xInv=1.28155157</pre>	<pre>invNorm area:0.90 μ:0 σ:1 Paste T184</pre>	<pre>invNorm(0.90,0,1) .....) 1.281551567</pre>
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- 7) The times for completing one circuit of a bicycle course are normally distributed with a mean of 68.0 minutes and a standard deviation of 4.9 minutes. An association wants to sponsor a race but will cut the bottom 25% of riders. In a trial run, what should be the cutoff time?

**Answer:** To find the cutoff time, you need to find the 25<sup>th</sup> percentile of the normal distribution. Locate 0.2500 in the body of the table. The corresponding z-score value in this case is  $-0.67$ , since 0.2514 is the closest value to 0.2500.

Solving for  $x$  in the formula for the z-score:  $x = \mu + z \cdot \sigma = 68 + (-0.67)(4.9) = 64.7$  minutes.

**On Calculators, Inv Normal:**

<pre> Inverse Normal Data :Variable Tail :Left Area :0.25 σ :4.9 μ :68 Save Res:None None LIST CASIO 9750 </pre>	<pre> Inverse Normal xInv=64.6950002 </pre>	<pre> InvNorm area:.25 μ:68 σ:4.9 Paste T184 </pre>	<pre> invNorm(.25,68,4.9) ..... 64.69500023 </pre>
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- 8) A bottler of drinking water fills plastic bottles with a mean volume of 999 milliliters (mL) and standard deviation 7 mL. The fill volumes are normally distributed. What is the probability that a bottle has a volume greater than 992 mL?

$$z = \frac{x - \mu}{\sigma} = \frac{992 - 999}{7} = -1 \quad \therefore P(x > 992) \Rightarrow 1 - P(z < -1) = 1 - 0.1587 = 0.8413$$

-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867

**On Calculators:**

<pre> Normal C.D Data :Variable Lower :992 Upper :1E+99 σ :7 μ :999 Save Res:None None LIST CASIO 9750 </pre>	<pre> Normal C.D P =0.84134474 z:Low=-1 z:UP =1.4286E+98 </pre>	<pre> normalcdf lower:992 UPPER:E99 μ:999 σ:7 Paste T184 </pre>	<pre> normalcdf(992,E99,999,7) ..... .8413447404 </pre>
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- 9) In order to be accepted into a certain top university, applicants must score within the top 5% on the SAT exam. Given that the exam has a mean of 1000 and a standard deviation of 200, what is the lowest possible score a student needs to qualify for acceptance into the university?

To the left of the top 5% we have the bottom 95%. Since tables includes areas to the left, we need to locate 0.9500 in the table's body. It corresponds to a z-score of 1.64.

Therefore,  $x = \mu + z \cdot \sigma = 1000 + 1.64(200) = 1328$

1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

<pre> Inverse Normal Data :Variable Tail :Left Area :0.95 σ :200 μ :1000 CASIO 9750 </pre>	<pre> Inverse Normal xInv=1328.97073 </pre>	<pre> InvNorm area:.95 μ:1000 σ:200 Paste T184 </pre>	<pre> invNorm(.95,1000,200) ..... 1328.970725 </pre>
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