Practice14, ANSWERS

Question 1: Find the mean of the distribution:

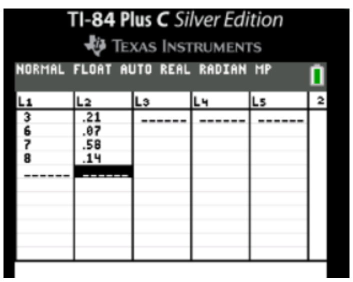
X	Px
3	0.21
6	0.07
7	0.58
8	0.14

<u>Answer</u>:

$\mu = \sum(x \times p(x))$ = $3\,(0.21)\ +\ 6(0.07)\ +\ 7(0.58)\ + 8(0.14)\ =\ 6.23$

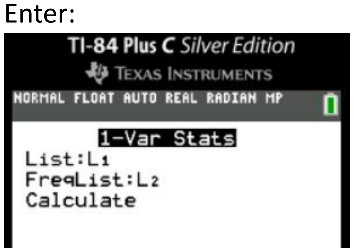
Or on graphing calculators:

On TI84 Plus, Press STAT enter X on $L1$ and $P(x)$ on $L2$

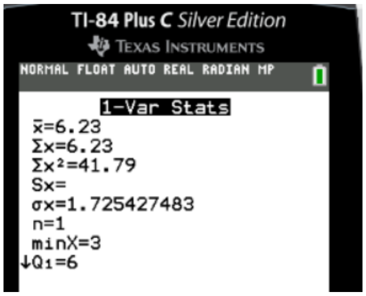


Press STAT again, choose CALC using navigation keys:

Enter:

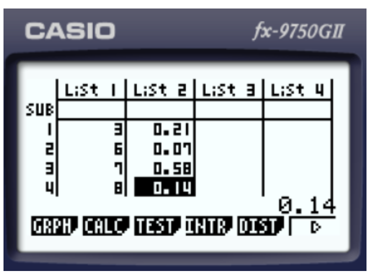


Calculate: press enter.
Results:

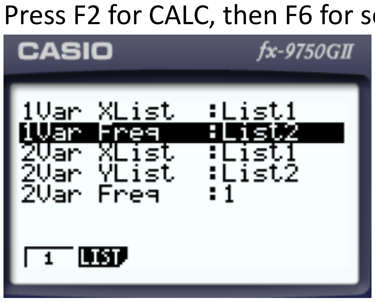


Mean is $\bar{x} = 6.23$
Notice that σx, the standard deviation $=$ 1.725427483
The Variance, if needed, is equal to the square of the standard deviation:
$\sigma^2 = 2.977$

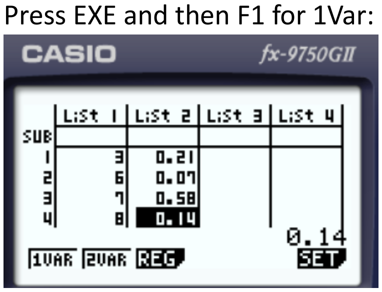
On CASIO 9750 GII
In STAT mode, enter X on $L1$ and $P(x)$ on $L2$



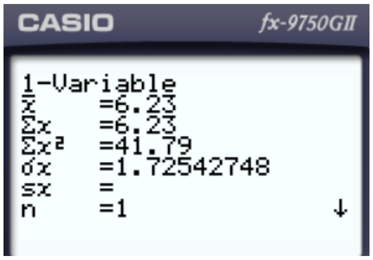
Press F2 for CALC, then F6 for set and make sure that 1Var Freq is set to List2:



Press EXE and then F1 for 1Var:



Results:



Mean is $\bar{x} = 6.23$
Notice that σx, the standard deviation $= \ 1.72542748$
The Variance, if needed, is equal to the square of the standard deviation:
$\sigma^2 = 2.977$

Question 2. Give the variance of the following distribution:

x	Px
0	0.20
1	0.35
2	0.10
3	0.25
4	0.10

<u>Answer:</u> Using the formula:

$$\sigma^2 = \sum [X^2 \cdot P(x)] - \mu^2$$

We need the mean, μ:

$$\mu = \sum\big(x \times p(x)\big) = 0\ (0.20)\ +\ 1(0.35)\ +\ 2(0.10)\ + 3(0.25)\ + 4(0.10) = 1.70$$

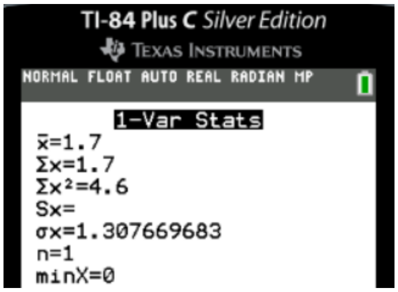
Create a table of $x, x^2, (x^2 \cdot Px)$:

x	x^2	Px	$X^2 \cdot Px$
0	0	0.20	0
1	1	0.35	0.35
2	4	0.10	0.40
3	9	0.25	2.25
4	16	0.10	1.6

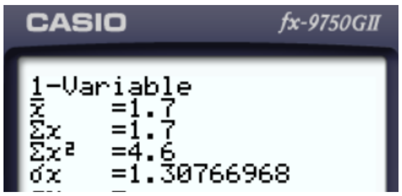
$$\sigma^2 = \sum [X^2 \cdot P(x)] - \mu^2 = [0 + 0.35 + 0.40 + 2.25 + 1.6] - 1.70^2$$

$$\sigma^2 = 4.6 - 2.89 = 1.71$$

On a graphing Calculator, after following the steps on question 1, the final results are:
On TI84:



Since $\sigma x = 1.307669683$, squaring this result, the variance, $\sigma^2 = 1.71$
Similarly on the Casio 9750GII, same output, same procedure:



Question 3. Compute the standard deviation:

x	$P(x)$
0	0.11
1	0.64
2	0.13
3	0.10
4	0.02

Answer:

Standard deviation formula:

$$\sigma = \sqrt{\sum[x^2 \cdot Px] - \mu^2}$$

First, find the mean:

$$\mu = \sum(x \cdot Px) = 0(0.11) + 1(0.64) + 2(0.13) + 3(0.10) + 4(0.02) = 1.28$$

Now we are ready to apply the formula for the standard deviation, σ:

$$\sigma = \sqrt{\sum[x^2 \cdot Px] - \mu^2} = \sqrt{[0 \cdot 0.11 + 1 \cdot 0.64 + 4 \cdot 0.13 + 9 \cdot 0.10 + 16 \cdot 0.02] - 1.28^2} = 0.8611\ldots$$

Instead of using the formula you may use a graphing calculator and follow the steps on question #2.

IMPORTANT! For questions 4, 6, 7 and 8 read the notes Calculating Expectations in Games on this page.

Question 4:
If a gambler rolls two dice and gets a sum of 10, he wins $10, and if he gets a sum of three, he wins $20. The cost to play the game is $5. What is the expectation of this game?

Answer: On two dice, there are three ways of getting a sum of ten: $(4,6)$ $(6,4)$ and $(5,5)$; that is, four on the first die and sic on the second die, or vice versa: six on the first die and four on the second, or five on both dice. There is no other way we can obtain s sum of ten when rolling two dice. Therefore, since there are $6\ x\ 6$ possible outcomes when rolling two dice (6 for the first die, times 6 for the second die) the prob of a sum of ten is $P_{sum=10} = \frac{3}{36}$

Probability of a sum = 3 occurs only in two ways: 1 in the first die and 2 in the second; or 2 ion the first and 1 in the second die, that is: (1,2) (2,1)

$$P_{sum=3} = \frac{2}{36}$$

In this scenario the gambler has a probability of winning either $10 or $20 of:

$$P_{sum=10} + P_{sum=3} = \frac{3}{36} + \frac{2}{36} = \frac{5}{36}$$

$$P_{losing} = 1 - P_{winning} = 1 - \frac{5}{36} = \frac{31}{36}$$

Now we are ready to complete a table:

X	$P(x)$
$10 - 5$	$3/36$
$20 - 5$	$2/36$
-5	$31/36$

Simplifying it reduces to:

X	$P(x)$
5	$1/12$
15	$1/18$
-5	$31/36$

Expected value, E(X), like the mean is found as follows:

$$E(X) = \sum(X \cdot P(X)) = 5(1/12) + 15(1/18) + -5(31/36) = -3.055 \approx -3.06$$

Question 5. The contractor makes $2700 profit per job. He may have up to four jobs per month. The discrete probability distribution of the number of jobs is given in a table as follows:

Number of Jobs	Probability
1	0.1
2	0.2
3	0.5
4	0.2

Answer:
Find the contractor expected profit per month.
In order to answer this question, we need to find the average (mean) number of jobs per month based on the given probability distribution and then multiply the average number of jobs times the amount the contractor makes per job ($2700).

Formula for the mean of a discrete probability distribution:
$\mu = \sum(x \times p(x))$
$\mu = 1\,(0.1) + 2(0.2) + 3(0.5) + 4(0.2) = 2.80$

Total profit $= 2.80 \times 2700 = \$7{,}560$

Or use a graphing calculator and enter number of jobs in L1 and probabilities on L2. Follow procedure on question 1.

Question 6. If a person rolls doubles when she tosses two dice, she wins $50. For the game to be fair, how much should she pay to play the game?

Answer**:** To be fair, the expectation must be zero. Using X as cost of playing, we have:
Winning gain: $50 - X$
Prob of winning: 6/36 (There are six doubles, in a total of 36 outcomes when rolling two dice)
Prob of losing: $1 - 6/36 = 30/36$

Reducing fractions to the lowest terms, the expectation equation becomes:
$(50 - X) \cdot \frac{1}{6} - X \cdot \frac{5}{6} = 0$ multiplying by 6 both sides becomes,

$(50 - X) - 5X = 0$ ∴ $50 - 6X = 0$ or $x = \frac{50}{6} = 8.33$

Question 7:

A lottery offers one $1000 prize, one $500 prize, and two $50 prizes. One thousand tickets are sold at $2.50 each. Find the expectation if a person buys one ticket.

Answer: As recommended on the step-by-step guide, use a table to organize calculations for each outcome:

Winning gain	Probability
1000 – 2.50	1/1000
500 – 2.50	1/1000
50 – 2.50	1/1000
50 -2.50	1/1000
Losing amount	
-2.50	996/1000

Simplifying:

x	P(x)
997.50	0.001
497.50	0.001
47.50	0.001
47.50	0.001
−2.50	0.996

Either using a graphing calculator or the mean (expected value) formula. It yields:

$$\mu = \sum [x \times P(x)]$$

$$= 997.50 \times 0.001 + 497.50 \times 0.001 + 47.50 \times 0.001 + 47.50 \times 0.001 - 2.50 \times 0.996 = -0.90$$

Question 8:
A lottery offers one $1000 prize, one $500 prize, and four $200 prizes. One thousand tickets are sold at $3.00 each. Find the expectation if a person buys two tickets.

Answer:
Create a table as follows:

Winning gain	Probability
1000 – 3	1/1000
500 – 3	1/1000
200 – 3	1/1000
200 – 3	1/1000
200 – 3	1/1000
200 – 3	1/1000
Losing amount	
-3	994/1000

The table can be reduced to:

X	$P(x)$
997	0.001
497	0.001
197	0.001
197	0.001
197	0.001
197	0.001
−3	0.994

Use the mean (expected value formula) or a graphing calculator to find the expected value for one ticket: -0.70; now, since the person buys two tickets the expected value will be twice that amount: $-\$1.40$