Confidence Intervals:

Based on a sample, calculate the interval of values in which the population parameter (mean, proportion, standard deviation) may lie with a degree of certainty. This degree of certainty (level of confidence) is given as a percentage or the equivalent decimal.
The most used confidence intervals are $80\%,\ 90\%,\ 95\%,\ 98\%\ and\ 99\%$.

The population parameter lies in between the sample point estimate plus or minus an error:

$$Sample\ point\ estimate\ \pm Error$$

The three confidence intervals in STA2023:

1. Confidence Interval for the population mean, population standard deviation σ, known.

$\bar{x} \pm Error$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

2. Confidence Interval for the population mean, population standard deviation σ, unknown. Use the sample standard deviation (for a normal distributed variable), use the $t\ distribution$.

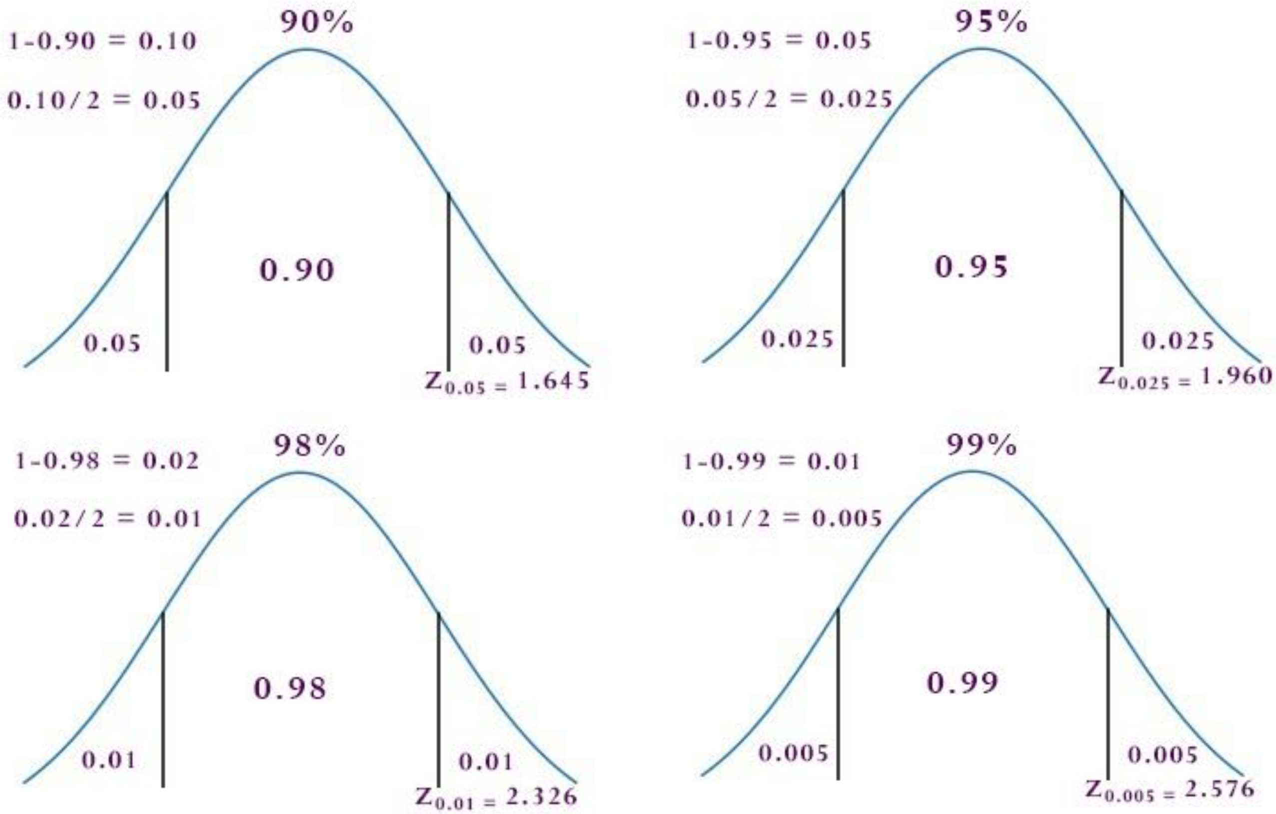
$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

3. Confidence Interval for the population proportion:

$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}$ where $\hat{q} = 1 - \hat{p}$

Sample size for means:	Sample size por proportions:
$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$	$n = \hat{p} \cdot \hat{q} \left(\frac{z_{\alpha/2}}{E}\right)^2$ If $\hat{p}$ unknown, use 0.5

Critical values, $Z_{\alpha/2}$



In short:

90%, $Z_{\alpha/2} = 1.645$ 95%, $Z_{\alpha/2} = 1.960$ 98%, $Z_{\alpha/2} = 2.326$ 99%, $Z_{\alpha/2} = 2.576$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	99%CI	98%CI	95%CI	90%CI	80%CI
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356

See complete table at the end of this document.

Degrees of freedom, $df = n - 1$ where n is the sample size.

Examples:

Question 1

The lengths, in inches, of adult corn snakes are normally distributed with a population standard deviation of 8 inches and an unknown population mean. A random sample of 25 snakes is taken and results in a sample mean of 58 inches. Find a 99% confidence interval estimate for the population mean.

$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $\quad 58 \pm 2.576 \cdot \frac{8}{\sqrt{25}}$ $\quad 58 \pm 4.1216$

by subtracting and adding $E = 4.1216$, yields the 99% Conf Interval: $(53.88, 62.12)$

Interpretation:

We can estimate with 99% confidence that the true population mean length of adult corn snakes is between 53.88 and 62.12 inches.

Question 2

Suppose the scores of a standardized test are normally distributed. If the population standard deviation is 3 points, what minimum sample size is needed to be 90% confident that the sample mean is within 2 points of the true population mean?

Sample size,

$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$ Therefore, $n = \left(\frac{1.645 \cdot 3}{2}\right)^2 = 6.08\ rounded\ (always)\ up = 7$

Question 3

The weekly salaries of sociologists in the United States are normally distributed and have a known population standard deviation of 425 dollars and an unknown population mean. A random sample of 22 sociologists is taken and gives a sample mean of 1520 dollars.

Find the margin of error for the confidence interval for the population mean with a 98% confidence level.

ME, margin of error given by $z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ Therefore, ME = $2.326 \cdot \frac{425}{\sqrt{22}} = 210.76$

Question 4

The commute times for workers in a city are normally distributed with an unknown population mean and standard deviation.

If a random sample of 20 workers is taken and results in a sample mean of 21 minutes and sample standard deviation of 6 minutes, find a 95% confidence interval estimate for the population mean using the Student's t-distribution.

df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
...	...	...	...	...	...
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831

Find the margin of error, for a 95% confidence interval estimate for the population mean using the Student's t-distribution.

$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ Margin of error, ME given by $t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

Therefore ME = $2.093 \cdot \frac{6}{\sqrt{20}} = 2.8081 = 2.81$

Interval, $\bar{x} \pm Error$ or 21 ± 2.81

by subtracting and adding E, yields the 95% Confidence interval: $(18.19, 23.81)$

Question 5

Emma wants to estimate the percentage of people who use public transportation in a city. She surveys 140 individuals and finds that 62 use public transportation. Find the confidence interval for the population proportion with a 99% confidence level.

Answer: This a proportion interval. Find $\hat{p}$ first:

$\hat{p} = \frac{x}{n} = \frac{62}{140} = 0.4429$ and $\hat{q} = 1 - \hat{p}$ Therefore, $\hat{q} = 1 - 0.4429 = 0.5571$

$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$ substituting values, becomes:

$$0.4429 \pm 2.576 \cdot \sqrt{\frac{0.4429 \text{ x } 0.5571}{140}}$$

$$0.4429 \ \pm 0.1081$$

$$(0.335, \qquad 0.551)$$

Question 6

Suppose a clothing store wants to determine the current percentage of customers who are over the age of forty.

How many customers should the company survey in order to be 90% confident that the estimated (sample) proportion is within 4 percentage points of the true population proportion of customers who are over the age of forty?

Answer: Sample size por proportions:

$n = \hat{p} \cdot \hat{q} \left(\frac{z_{\alpha/2}}{E}\right)^2$ since $\hat{p}$ is unknown (some times researchers used a previous study result, that is the case in which $\hat{p}$ is known; otherwise, like in this case, use 0.5; the $z_{\alpha/2}$ = 1.645 for a 90% CI, 4% as decimal, $E = 0.04$; substituting values:

$$n = 0.5 \cdot 0.5 \left(\frac{1.645}{0.04}\right)^2 = 422.81 = 423$$

TABLE A-3 *t* Distribution: Critical *t* Values

Degrees of Freedom	Area in One Tail 0.005	0.01	0.025	0.05	0.10
	99%CI	98%CI	95%CI	90%CI	80%CI
	Area in Two Tails 0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356
13	**3.012**	2.650	**2.160**	1.771	1.350
14	**2.977**	2.624	**2.145**	1.761	1.345
15	**2.947**	2.602	**2.131**	1.753	1.341
16	**2.921**	2.583	**2.120**	1.746	1.337
17	**2.898**	2.567	**2.110**	1.740	1.333
18	**2.878**	2.552	**2.101**	1.734	1.330
19	**2.861**	2.539	**2.093**	1.729	1.328
20	**2.845**	2.528	**2.086**	1.725	1.325
21	**2.831**	2.518	**2.080**	1.721	1.323
22	**2.819**	2.508	**2.074**	1.717	1.321
23	**2.807**	2.500	**2.069**	1.714	1.319
24	**2.797**	2.492	**2.064**	1.711	1.318
25	**2.787**	2.485	**2.060**	1.708	1.316
26	**2.779**	2.479	**2.056**	1.706	1.315
27	**2.771**	2.473	**2.052**	1.703	1.314
28	**2.763**	2.467	**2.048**	1.701	1.313
29	**2.756**	2.462	**2.045**	1.699	1.311
30	**2.750**	2.457	**2.042**	1.697	1.310
31	**2.744**	2.453	**2.040**	1.696	1.309
32	**2.738**	2.449	**2.037**	1.694	1.309
33	**2.733**	2.445	**2.035**	1.692	1.308
34	**2.728**	2.441	**2.032**	1.691	1.307
35	**2.724**	2.438	**2.030**	1.690	1.306
36	**2.719**	2.434	**2.028**	1.688	1.306
37	**2.715**	2.431	**2.026**	1.687	1.305
38	**2.712**	2.429	**2.024**	1.686	1.304
39	**2.708**	2.426	**2.023**	1.685	1.304
40	**2.704**	2.423	**2.021**	1.684	1.303
45	**2.690**	2.412	**2.014**	1.679	1.301
50	**2.678**	2.403	**2.009**	1.676	1.299
60	**2.660**	2.390	**2.000**	1.671	1.296
70	**2.648**	2.381	**1.994**	1.667	1.294
80	**2.639**	2.374	**1.990**	1.664	1.292
90	**2.632**	2.368	**1.987**	1.662	1.291
100	**2.626**	2.364	**1.984**	1.660	1.290
200	**2.601**	2.345	**1.972**	1.653	1.286
300	**2.592**	2.339	**1.968**	1.650	1.284
400	**2.588**	2.336	**1.966**	1.649	1.284
500	**2.586**	2.334	**1.965**	1.648	1.283
1000	**2.581**	2.330	**1.962**	1.646	1.282
2000	**2.578**	2.328	**1.961**	1.646	1.282
Large	**2.576**	2.326	**1.960**	1.645	1.282
	99%	98%	95%	90%	80%

large n
t becomes z