A **variable** is defined as a characteristic or attribute that can assume different values. A random variable is a variable whose values are determined by chance.

Discrete variables have a finite number of possible values or an infinite number of values that can be counted.

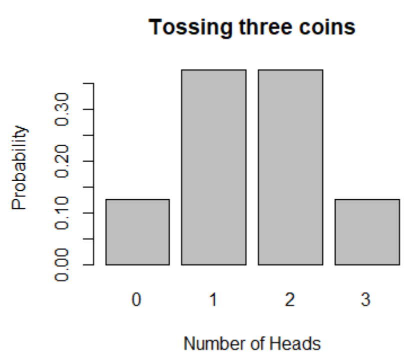
A **discrete probability distribution** consists of the values a random variable can assume and the corresponding probabilities of the values. The probabilities are determined theoretically or by observation.

Example: Tossing Coins

Represent graphically the probability distribution for the sample space for tossing three coins.
The sample space consists of **8** possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Probability [$P(x)$] distribution of **number of heads** (x) **when tossing three coins:**

x	$P(x)$
0	$1/8$
1	$3/8$
2	$3/8$
3	$1/8$



Number of children (under 18) living in household, USA 2022 (source: statista.com)

x	$p(x)$
0	0.597
1	0.171
2	0.148
$3\ or\ more$	0.083

<u>Two Requirements for a Probability Distribution</u>:

1. The sum of the probabilities of all the events in the sample space must equal 1; that is,

$$\sum P(x) = 1.$$

2. The probability of each event in the sample space must be between or equal to 0 and 1.

That is, $0 \leq P(x) \leq 1$

5.2 Mean, Variance, Standard Deviation, and Expectation

Formula for the **Mean** of a Probability Distribution: $\mu = \sum x \cdot p(x)$

Variance of a probability distribution:

$\sigma^2 = \sum[(x - \mu)^2 \cdot p(x)]$ which is equivalent to:

$$\sigma^2 = \sum \left(x^2 \cdot p(x)\right) - \mu^2$$

The standard deviation of a probability distribution is

$$\sigma = \sqrt{(\sigma^2)}$$

Expectation

Another concept related to the mean for a probability distribution is that of expected value or expectation. Expected value is used in various types of games of chance, in insurance, and in other areas, such as decision theory.

The expected value of a discrete random variable of a probability distribution is the theoretical average of the variable. The formula is

$$\mu = E(X) = \sum X \cdot P(X)$$

The symbol $E(X\ )$ is used for the expected value.

The formula for the expected value is the same as the formula for the theoretical mean. The expected value, then, is the theoretical mean of the probability distribution.

That is, $E(X) = \mu$

Expectation in games:

To calculate the expected value (expectation) in a game, you can follow these steps:

Identify the possible outcomes: List all possible outcomes of the game, including both winning and losing scenarios. Each outcome should have a clear gain or loss associated with it.

Assign probabilities: Determine the probability of each outcome occurring. This might involve using known probabilities (e.g., dice rolls), historical data (e.g., win rates), or even subjective estimates based on your understanding of the game.

Calculate gain/loss per outcome: Multiply the gain or loss associated with each outcome by its corresponding probability. This gives you the "weighted value" of each outcome.

Sum the weighted values: Add up the weighted values from all possible outcomes. This final sum represents the **expected value** of the game.

Expected value represents the average outcome over many plays, not necessarily a single game.

5.3 The Binomial Distribution

Many types of probability problems have only two outcomes or can be reduced to two outcomes. For example, when a coin is tossed, it can land heads or tails. When a baby is born, it will be either male or female. In a basketball game, a team either wins or loses. A true/false item can be answered in only two ways, true or false.

A binomial experiment is a probability experiment that satisfies the following four requirements:

1. There must be a fixed number of trials.
2. Each trial can have only two outcomes or outcomes that can be reduced to two outcomes. These outcomes can be considered as either success or failure.
3. The outcomes of each trial must be independent of one another.
4. The probability of a success must remain the same for each trial.

The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a binomial distribution.

In a binomial experiment, the probability of exactly X successes in n trials is

$$p(x) = nCx \cdot p^x \cdot q^{n-x}$$

Where $\boldsymbol{n}$ is the total number of trials, $\boldsymbol{x}$ is the number of successes, $\boldsymbol{p}$ is the probability of success and $\boldsymbol{q}$ is the probability of failure. Since there are only two possible outcomes, $p + q = 1$.

Example 1: A coin is tossed 10 times. Find the probability of getting exactly seven heads.
$n = 10,\ x = 7,\ p = 1/2,\ q = 1/2$

$p(7) = 10C7 \cdot p^7 \cdot q^3 \quad = \quad 0.1171875$

The mean, variance, and standard deviation of a variable that has the binomial distribution can be found by using the following formulas.

Mean: $\mu = n \cdot p$ Variance: $\sigma^2 = n \cdot p \cdot q$

Standard deviation: $\sigma = \sqrt{n \cdot p \cdot q}$

Calculating the mean, variance and standard deviation for example 1:

a. Mean: $\mu = n \cdot p \quad = \quad 10 \times 0.5 \quad = 5$

b. Variance: $\sigma^2 = n \cdot p \cdot q \quad = \quad 10 \times 0.5 \times 0.5 = 2.5$

c. Standard déviation : $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{2.5} \approx 1.58$