Normal distribution using formulas:

1. Given the population mean, $\mu = 32$; and, the population standard deviation, σ = 2.25, find:
   a) Probability of $x < 30$.
   b) Probability of $x > 35$.
   c) Probability of x greater than 30 and less than 35: $P_{30<x<35}$
   d) If we choose 56 values of the random variable at random, and the sample mean is = 33, considering that the population standard deviation is 2.25, what is the probability that samples of the same size are less than 33?
   e) What is the x value that is above 99% of all other values of the variable?

Answers:

a) $P(x < 30) = \frac{x-\mu}{\sigma} = \frac{30-32}{2.25} = -0.8888 \ldots \approx -0.89$

Now, $P(z < -0.89) = 0.1867$ by the table:

−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776

b) Probability of $x > 35$.

$$P(x > 35) = \frac{x-\mu}{\sigma} = \frac{35-32}{2.25} = 1.333 \ldots \approx 1.33$$

1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

$P(z > 1.33) = 1 - P(z < 1.33) = 1 - 0.9082 = 0.0918$

c) Probability of x greater than 30 and less than 35: $P_{30<x<35}$

$P(x < 30) = \frac{x-\mu}{\sigma} = \frac{30-32}{2.25} = -0.8888 \ldots \approx -0.89$ & $P(z < -0.89) = 0.1867$

$P(x > 35) = \frac{x-\mu}{\sigma} = \frac{35-32}{2.25} = 1.333 \ldots \approx 1.33$ & $P(z < 1.33) = 0.9082$

Therefore,

$P_{30<x<35} = 0.9082 - 0.1867 = 0.7215$

d) If we choose 56 values of the random variable at random, and the sample mean is = 33, considering that the population standard deviation is 2.25, what is the probability that samples of the same size are less than 33?

$$P(\bar{x} < 33) = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{33 - 32}{2.25/\sqrt{56}} = 3.3259 \approx 3.33$$

3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$$P(z < 3.33) = 0.9996$$

e) What is the x value that is above 99% of all other values of the variable?

Find on the table the z score that corresponds to an area of 99% or 0.99:

2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

The z score is 2.33:

On the formula for the z-score,

$$z = \frac{x - \mu}{\sigma}$$

Solving for x,

$$x = z \cdot \sigma + \mu$$

Substituting values:

$$x = 2.33 \cdot 2.25 + 32 = 37.24$$

The answer to e) is the variable x value that is above 99% of the population is $x = 37.24$, rounding to two decimal places.