Answers to Hypothesis testing examples:

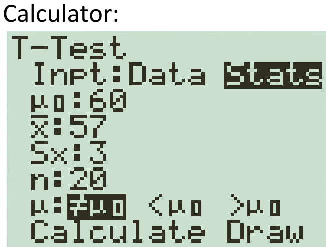
Note: In the first three problems, we test a claim about a population by using one sample data.
In the second part, problems IV to VI, we compare two populations using two samples data.
Additionally, we construct the corresponding confidence intervals: for one sample problems, when the confidence interval include the population parameter stated on the Null Hypothesis, implies that the samples belongs to the given population and therefore we fail to reject the Null; on the other hand, if the confidence interval does not include the population parameter, it suggests that the sample data describes a different population, so we reject the Null hypothesis.
For two samples intervals, whenever the interval negative and positive values, we say that the interval includes zero: it suggests that there is no difference between the two populations' parameters. If the interval includes only negative values or only positive values, it implies that the two population's parameters differ: one is always larger than the other (we reject the hypothesis of equality between the two populations: the Null.)

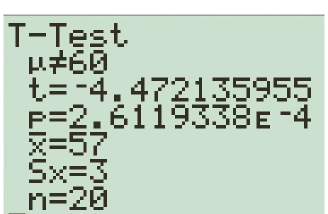
I. H_0: $\mu = 60$
H1: $\mu \neq 60$

Note: the claim states a value; Temp (..) is equal to 60; it does not say greater or less, just a value. In this case the alternative is not equal to (≠).
The sample mean and standard deviations are given; therefore, this is a T-Test.
Level of significance is 0.01 ($\alpha = 0.01$); because the alternative hypothesis uses ≠, the test is a two tailed test. The t(alpha/2) critical value, for 20-1 = 19 degrees of freedom by the T-Table is 2.861. Remember, the critical value is needed in order to establish our conclusions, when we don't use a graphing calculator: we compare the T-test statistic, which we are calculating below, to the critical value: when the absolute value of the test-test is greater than the critical value we Reject the Null hypothesis. Let's find the test stat:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{57 - 60}{\frac{3}{\sqrt{20}}} = -4.47$$

Calculator:

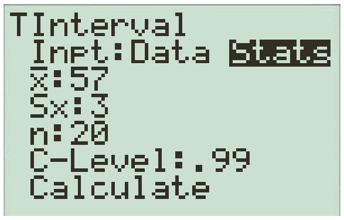


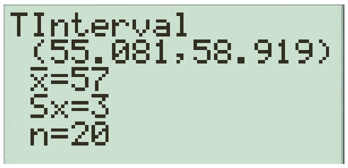


The output of the t-test shows a t-statistic, t = -4.47 and a p-value = 0.00026 or 2.6 x 10^{-4}
Confidence interval: For a two=-tailed test, construct the 1- α conf. int: in this case 1-0.01 = 0.99

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 57 \pm 2.861\frac{3}{\sqrt{20}} = (55.1, 58.9)$$

Calculator: T-Int:





Conclusions: Reject the Null Hypothesis. There are two ways of drawing this conclusion: the t-stat absolute value is greater than the critical value or, the p-value is less than the stated value of alpha.
In technical terms: we have enough sample evidence to warrant rejection of the claim that the mean temperature is 60 degrees Fahrenheit.
Notice that the interval (55.1, 58.9) does not include the stated population parameter of 60.
That fact is consistent with the conclusion of rejecting H0: the sample describes a population whose mean is between 55.1 and 58.9, range of values that exclude 60.

II

H_0: p = 0.60
H1: p > 0. 60

One proportion Z-Test.
α = 0.05; right tailed test.
$Z_{0.05}$= 1.645

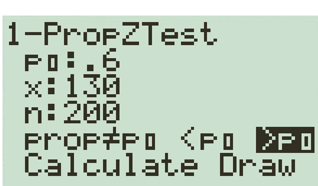
First, calculate the sample proportion:

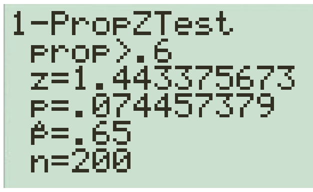
$$\hat{p} = \frac{x}{n} = \frac{130}{200} = 0.65$$

Z- test Statistic:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.65 - 0.60}{\sqrt{\frac{0.60 * 0.40}{200}}} = 1.44$$

Calculator:



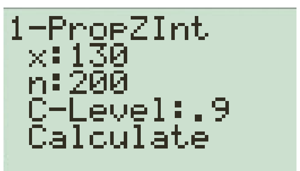


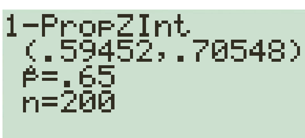
Calculator: test- statistic z =1.44; p-value = 0.074

Confidence interval:
This is a one tailed test, we construct the 1-2α conf. interval. This is the reason why: intervals include two tails, whenever the test is a one-tailed test, we need to *create* the other tail; so we subtract two times alpha from one. In this case we construct a 1-2(0.05) = 0.90 Conf. Int.

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.65 \pm 1.645\sqrt{\frac{0.65 * 0.35}{200}} = (0.59, 0.71)$$

1-Prop ZInt by Calculator:





Notice that the z-test statistic (1.44) is less than the critical value (1.645); also, the p-value, 0.074 > alpha (0.01).

Conclusions: We fail to reject the Null hypothesis.
In technical terms: there is no sufficient evidence to support the claim that over 60% of the citizens approve the mayor's job.

The confidence interval includes 0.60 (0.59, 0.71); that is, the sample data is consistent with the Null hypothesis; therefore, the is no support for the claim that indeed the proportion is greater than 60%.

III.

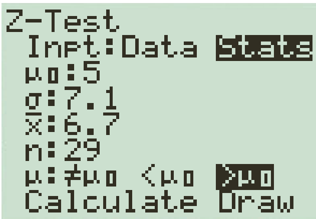
H_0: μ = 5
H1: μ > 5

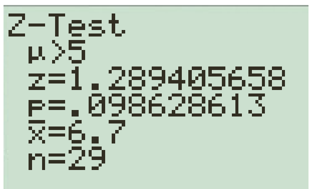
Significance level, α =0.05; right tailed test.
Since the problem states that *the standard deviation of all women was 7.*1; that is, sigma (σ) is given, so we use a Z-Test.
The $z_{0.05}$ = 1.645, which is the critical value.

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{6.7 - 5}{\frac{7.1}{\sqrt{29}}} = 1.29$$

Calculator: Z-Test:

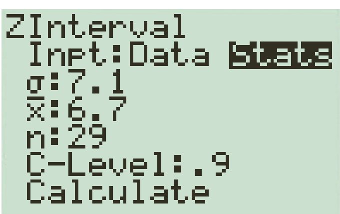


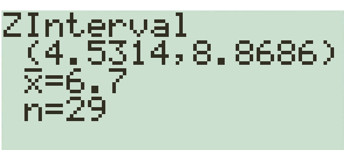


The 1-2alpha CI: 1-2(0.05) = 0.90

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 6.7 \pm 1.645\frac{7.1}{\sqrt{29}} = (4.5, 8.9)$$

Calculator: Z-Int: 90% Conf. Int:





Conclusions: Fail to reject the Null hypothesis. According to the sample data there is no sufficient evidence to support the claim that the average weight gain per woman was over five pounds.
Notice that the confidence interval includes five; therefore, the true value of the population parameter may be five, not a value significantly greater than five. This fact is consistent with the decision of not rejecting the Null hypothesis ($\mu = 5$).

IV. 2 Samples T Test

H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 > \mu_2$

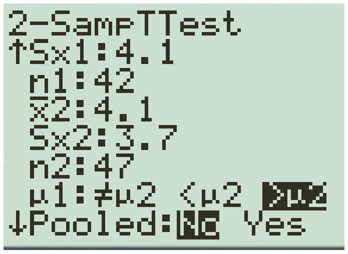
The question (claim) is: Does dieters lose more fat than the exercisers? Since we label first group (dieters) as 1, and exercisers as 2, the alternative hypothesis is symbolized: $\mu_1 > \mu_2$.

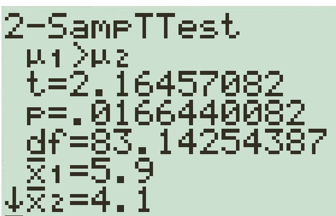
It is a right tailed test; the critical value is t = 2.423. It is the corresponding value for alpha = 0.01 and 41 df – degrees of freedom: smaller n-1. Notice that graphing calculators and statistical software generate different degrees of freedom by using Satterthwaite formula. Our textbook, Elementary Statistics by M. Triola, follows the rule smaller n-1. One method or the other does not change the final conclusion.

Test Statistic: We use this formula when the populations' variances are different for both groups. In the calculator menu this is achieved by choosing pooled No. This is called the Welch T test, the test by default in statistical software like R.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{5.9 - 4.1}{\sqrt{\frac{4.1^2}{42} + \frac{3.7^2}{47}}} = 2.16$$

Calculator:

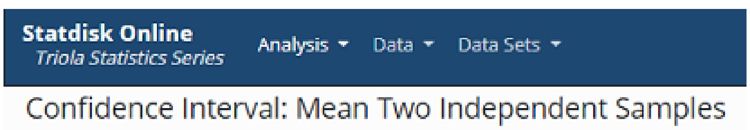




The output reads: t statistic = 2.16, p-value = 0.0166.

Again, we construct the 1-2α conf. interval: 1-2(0.01)=0.98

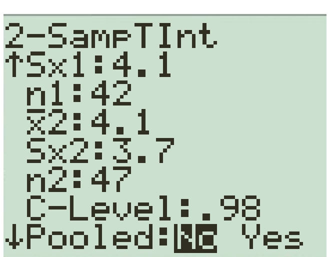
Software, statdisk.org:

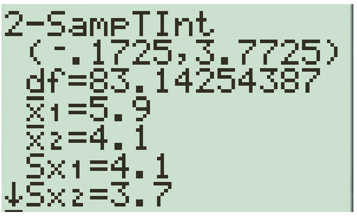


Statdisk.org Software output:

```
Test Statistic, t:  2.16457
Critical t:         ±2.37204
P-Value:            0.03329

Degrees of freedom: 83.14254

98% Confidence Interval:
-0.17253 < μ1-μ2 < 3.77253
```

Two samples T Interval. TI calculator:





Notice that the test statistic is less than the t critical value, and the p-value is larger than alpha; also, the confidence interval includes zero. All this information suggests that the two populations' means are equal.

Conclusions: We fail to reject the Null Hypothesis. There is no sufficient evidence to answer yes to the question that dieters lose more weight than exercisers.

V. 2 samples T Test.

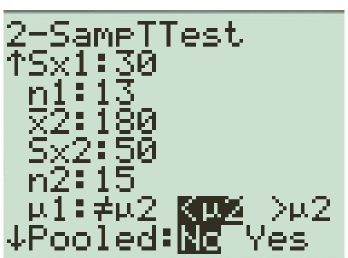
Label treatment group as 1; and control group as 2 (it could be the other way and we will draw the same conclusions): the researcher test whether the treatment is effective, in that case the mean systolic blood pressure of group 1 would be lower than the mean for group two.

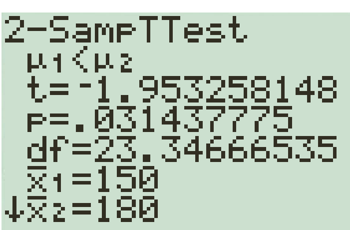
H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 < \mu_2$

Left tailed test, alpha = 0.01; t critical = -2.681 (see T-Table).
Test Statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{150 - 180}{\sqrt{\dfrac{30^2}{13} + \dfrac{50^2}{15}}} = -1.95$$

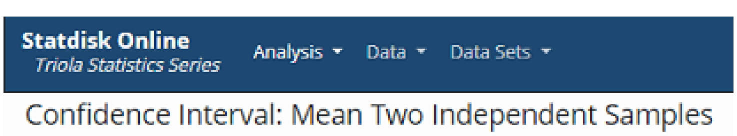
Calculator:





Test stat: t = -1.95; p-value: 0.0314

The 1-2α conf. interval: 1-2(0.01)=0.98
By Statdisk.org:

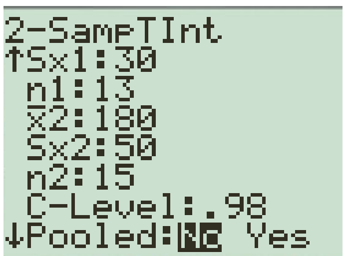


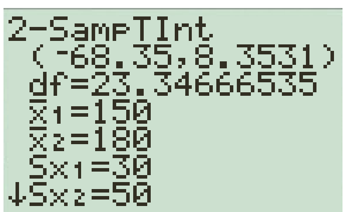
Statdisk.org output:

```
Test Statistic, t:  -1.95326
Critical t:         ±2.49712
P-Value:            0.06288

Degrees of freedom: 23.34667

98% Confidence Interval:
-68.35311 < μ1-μ2 < 8.35311
```

TI Calculator: 2 sample T interval:

(next page)





Notice that the test statistic absolute value is less than the critical value; accordingly, the p-value is of 0.0314 is greater than the stated significance level, and the confidence interval includes zero.

Conclusions: Fail to reject the Null. There is no sufficient evidence to support the researcher's claim that the treatment has been effective in lowering the systolic blood pressure.

VI: 2-Proportions Z Test:

H_0: $p_1 = p_2$
H_1: $p_1 \neq p_2$

The research consists of testing whether there is a difference or not between the two medications. That is the reason why we choose the symbols to be equal and not equal to (Is there a difference or not?).
This is a two-tailed test with significance level of 0.01; the Z critical value is 2.575.

Test Statistic:
First, calculate the two samples proportions using the formula:

$$\hat{p} = \frac{x}{n}$$

And p-bar as follows:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\hat{p_1} = 0.10; \hat{p_2} = 0.06; \bar{p} = 0.08$$

$$z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}=\frac{0.10-0.06}{\sqrt{0.08(0.92)\left(\frac{1}{200}+\frac{1}{200}\right)}}=1.47$$

TI Calculator:

```
2-PropZTest
 x1:20
 n1:200
 x2:12
 n2:200
 p1:≠p2 <p2 >p2
 Calculate Draw
```

```
2-PropZTest
 p1≠p2
 z=1.474419562
 p=.1403687297
 p̂1=.1
 p̂2=.06
↓p̂=.08
```

Test Stat is z=1.47; p-value = 0.1404
The 1-α conf. interval: 1-(0.01)=0.99
By Statdisk.org:

```
Statdisk Online    Analysis ▾   Data ▾   Data Sets ▾
 Triola Statistics Series
Confidence Interval: Proportion Two Samples

Test Statistic, z:  1.47442
Critical z:         ±2.57583
P-Value:            0.14037

99% Confidence interval:
-0.02969 < p1-p2 < 0.10969
```

TI Calculator:

```
2-PropZInt
 x1:20
 n1:200
 x2:12
 n2:200
 C-Level:.99
 Calculate
```

```
2-PropZInt
 (-.0297,.10969)
 p̂1=.1
 p̂2=.06
 n1=200
 n2=200
```

Notice that the test statistic is less than the critical value, and the p-value is greater than alpha; accordingly, the corresponding confidence interval includes zero.

Conclusions: Fail to reject the Null hypothesis. There is no sufficient evidence to determine that indeed there is a difference in the percentage of adult patients' reactions.