Counting rule, permutations, combinations?

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1. A debit card pin number consists of four digits –selected from 0 to 9. The customer is allowed to repeat digits. What is the probability that someone who does not know the pin number in use, guesses the correct one?

   In this problem there are 10 digits to choose from in order to select a PIN number.

   Two ways of answering this question:

   a. Since the number of choices for the first, the second, the third and fourth digits are 10, then there are $10 \times 10 \times 10 \times 10 = 10{,}0000$ number of choices of which only one is the correct pin number. The probability of guessing that number is $\frac{1}{10{,}000}$

   b. Since the PIN number has four digits, and the probability of guessing is constant and equal to $\frac{1}{10}$, the probability of guessing all four in a row is given by:

   $$P(A \text{ and } B \text{ and } C \text{ and } D) = P_A \cdot P_B \cdot P_C \cdot P_D$$

   $$= \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10{,}000}$$

   Notice that event A consists of "guessing the first digit correctly"; event B "guessing the second digit correctly"; event C "guessing the third digit correctly" and event D "guessing the fourth digit correctly"

2. From the ordinary deck of cards, 52 in total, what is the probability of choosing three and they are all Kings if every time a card is chosen, it is replace back to the original deck of cards.
   Answer: Like in the previous example, the total number of choices remain constant. The probability of choosing one King every time remains constant and equal to $\frac{4}{52}$ since there are four Kings in a deck of cards: $P(A \text{ and } B \text{ and } C) = P_A \cdot P_B \cdot P_C$

   Where event A consists of "choosing a card and it is a King", event B "choosing a second card and it is also a King", and event C "choosing a third card and it is also a King":

   Therefore,

   $$P(A \text{ and} B \text{ and } C) = P_A \cdot P_B \cdot P_C = \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \left[\frac{1}{13}\right]^4 = \frac{1}{2197}$$

3. Similar to question 1, but this time the digits on the PIN numbers cannot be repeated. So, now we are selecting 4 out of 10 digits but we cannot use the same digit more than once.
   Of course, we know that on a PIN number like in passwords, the order matters. So, in this case we may answer the question in two ways:

   a. For someone guessing the PIN number the probability of guessing the first one correctly is $\frac{1}{10}$; for the second one, $\frac{1}{9}$; for the third one, $\frac{1}{8}$, and for the fourth, $\frac{1}{7}$ --every time there is one digit less available.

   $$P(A \text{ and } B \text{ and } C \text{ and } D) = \frac{1}{10} \cdot \frac{1}{9} \cdot \frac{1}{8} \cdot \frac{1}{7} = \frac{1}{5040}$$

   b. Using permutations: since order of the digits in the PIN number is relevant, we use permutations: that is, out of 10 digits we select 4. Since only one of the four digits numbers is the correct PIN number, the probability of guessing the PIN is given by:

   $$P(\text{guessing the PIN number}) = \frac{1}{10P4} = \frac{1}{5040}$$

4. Similar to question 2, but this time cards, once selected are not replaced back to the original deck of cards:

   a. Answer using combinations, since the order in which the Kings are selected is irrelevant:
   $$P(\text{choosing } 3 \text{ Kings}) = \frac{4C3}{52C3} = \frac{4}{22100} = \frac{1}{5525}$$

   b. Using the multiplication rule:
   $P(A \text{ and } B \text{ and } C)$: where event A is selecting the first King; event B, selecting the second King, and event C selecting the third King:

$$P(A \text{ and } B \text{ and } C) = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{1}{5525}$$

Notice that when choosing the first card there are 4 Kings out of 52 ; when choosing the second, only three Kings left and in total there is one card less, 51; etc.

IN SHORT: We cannot use permutations or combinations if the total number of choices remains constant. However, if selecting an item removes it from the available pool, we use permutations when the order of the selected items matters, and combinations when the order does not matter.