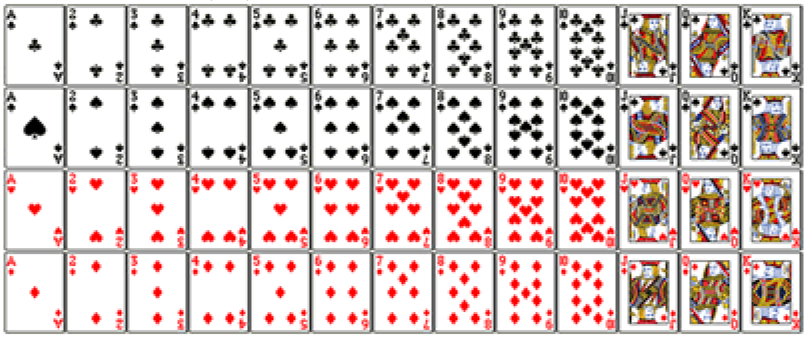
A deck of cards consists of 4 suits, 13 cards each for a total of 52 cards. The four suits are: Spades, Hearts, Diamonds, and Clubs. Each suit contains an A (Ace), numbers from 2 to 10, and three face cards: J, The Jack; Q, the Queen and K, the King.



QUESTIONS: Let's assume a well shuffled deck of card is available for every trial.
Find the probability of (in all these events you are choosing just one card):

1. Choosing a red or a black card.
2. Choosing a spade or a heart.
3. Choosing a King or an Ace.
4. Choosing a King or a spade.
5. Choosing a red card or a Queen.
6. Choosing an even numbered card or a black card.
7. Choosing and spade or an Ace.

Find the probability of (for the following events you are choosing more than one card; and, every time, before you choose a new card, put the chosen one back, and shuffle again):

8. Choosing two spades.
9. Choosing a spade and a heart.
10. Choosing three Aces in a row.
11. Choosing a heart, a King, a Jack in that order.

Find the probability of (for the following events you are choosing more than one card without replacement):

12. Choosing three spades in a row.
13. Choosing two hearts.
14. Choosing a King, a Queen and a Jack in that order.
15. Choosing five cards and they all are diamonds.

Find the probability of (in all these events you are choosing just one card, but a ***given*** condition is known):

16. Choosing a King given that is a black card.
17. Choosing a diamond given that is a red card.
18. Choosing a Queen given that is a face card.
19. Choosing and Ace given that is a face card.
20. Choosing a diamond given that is an Ace card.
21. Choosing a Queen given that the card is a spade.

Permutations and combinations:

22. In how many way you can choose 5 cards out of the 52 cards?
23. In how many ways you can you can arrange the three face cards of a given suit?

Find the probability of (without replacement):

24. Choosing 5 cards at random and they all are diamonds.
25. Choosing 3 cards such that one is a King and the two others are Aces.
26. Choosing three cards and they all are red cards.
27. Choosing 5 cards and getting 3 hearts and 2 clubs.
28. Choosing 5 cards and they are all Hearts.

Answers:

For mutually exclusive events: P(A or B) = P(A) + P(B)
For not mutually exclusive events, both events may occur at the same time (overlap or intersection):
P(A or B) = P(A) + P(B) – P(A and B)
For each question declare what is event A, what is considered event B:

1. A: choosing red card; B, choosing black card:
   P(A or B) = P(A) + P(B) =26/52 + 26/52 = 1.

2. A: choosing a spade; B: choosing a heart:
   P(A or B) = P(A) + P(B) = 13/52 + 13/52 =26/52 = ½

3. A: choosing a king; B: choosing an Ace:
   P(A or B) = P(A) + P(B) =4/52 + 4/52 = 8/52 = 2/13

4. A: choosing a king; B: choosing a spade.
   Since there is a king of spades, overlap exists. Events are not mutually exclusive:
   P(A or B) = P(A) + P(B) – P(A and B) = 4/52 + 13/52 – 1/52 = 16/52 =4/13

5. A: choosing a red card; B: choosing a queen.
   Since two of the red card cards include the queen of hearts (red), and the queen of diamonds (also red). Overlap exists.
   P(A or B) = P(A) + P(B) – P(A and B) =26/52 + 4/52 – 2/52 = 28/52 = 7/13

6. A: choosing an even numbered card; choosing a black card:
   Each suit includes five even numbered cards (2, 4, 6, 8, 10); and two of the suits are black cards. Overlap exists:
   P(A or B) = P(A) + P(B) – P(A and B) = 20/52 + 26/52 – 10/52 = 36/52 = 9/13

7. A: choosing and spade; B: choosing an Ace.
   There is an Ace of space; therefore, overlap or intersection exists:
   P(A or B) = P(A) + P(B) – P(A and B) = 13/52 + 4/52 – 1/52 = 16/52 = 4/13

8. A: choosing one spade; B, choosing another spade (with replacement):
   P(A and B) = P(A) * P(B) = 13/52*13/52 = 1/16

9. A: choosing one spade; B, choosing a heart (with replacement):
   P(A and B) = P(A) * P(B) = 13/52*13/52 = 1/16

10. A: choosing an Ace, B: choosing another Ace; C: choosing another Ace (with replacement):
    P(A and B and C)= P(A) * P(B) *P(C) = 4/52*4/52*4/52 = $(1/13)^3$ = 1/2197

11. A: choosing a heart, B: choosing a king; C: choosing a jack (with replacement):
    P(A and B and C)= P(A) * P(B) *P(C) = 13/52*4/52*4/52 = 1/676

12. A: choosing a spade; B, choosing another spade; C: choosing a third spade without replacement: P(A and B and C)= P(A) * P(B|A) * P(C|A and B) = 13/52 * 12/51 *11/50 = 11/850

13. A: choosing a heart; B, choosing another heart without replacement: P(A and B) = P(A) * P(B|A) = 13/52*12/51 = 1/17

14. A: choosing a king; B, choosing a queen; C: choosing a jack, without replacement: P(A and B and C)= P(A) * P(B|A) * P(C|A and B) = 4/52 * 4/51 *4/50 = 8/16575

15. A: choosing a diamond; B, choosing a diamond; C: choosing a diamond; D, choosing a diamond; E, choosing a diamond; without replacement: P(A and B and C and D and E) = P(A) * P(B|A) * P(C|A and B) * P(D|A and B and C) * P(E|A and B and C and D and E)= 13/52*12/51*11/50*10/49*9/48 = 99/204085

16. P(King|Black card) = 2/26 =1/13 [Given that the card is black, 26 of them, 2 of them are King and Black]
    Formal approach: P(King|Black) =$\frac{P(K\cap B)}{P(B)}$ = $\frac{\frac{2}{52}}{\frac{26}{52}} = 2/26 = 1/13$

17. P(Diamond|red card) = 13/26 =1/2

18. P(Queen|face card) = 4/12 =1/3

19. P(Ace|face card) = 0/12 = 0 [There are no Aces among face cards].

20. P(diamond|Ace) = 1/4 [There is one diamond among Ace cards]

21. P(Queen|spade) = 1/13 [there is one queen among spades]

22. $52C5 = 2598960$

23. $13P3 = 1716$

24. $\frac{13C5}{52C5} = \frac{33}{66640}$

25. $\frac{4C1 \cdot 4C2}{52C3} = \frac{6}{5525}$

26. $\frac{26C3}{52C3} = \frac{2}{17}$

27. $\frac{4C3 \cdot 13C2}{52C5} = \frac{1}{8330}$

28. $\frac{13C5}{52C5} = \frac{33}{66640}$