

Calculating Expectation in Games: Step-by-Step Guide

Here's how to calculate the expected value (average outcome) of a game when you know the winning probability and play cost:

Step 1: Define Outcome Values

- Identify the amount you **gain** if you win (positive value).
- Identify the amount you **lose** if you play (negative value, usually the cost of playing).

Step 2: Assign Probabilities

- Determine the probability of winning (expressed as a decimal between 0 and 1).
- The probability of losing is simply 1 minus the winning probability (1 - winning probability).

Step 3: Multiply Values by Probabilities

- Multiply the winning gain by the winning probability.
- Multiply the losing amount by the losing probability.

Step 4: Calculate Expected Value

- Add the products from Step 3. This represents the average outcome you can expect over many plays.

Formula:

Expected Value = (Winning Gain * Winning Probability) + (Losing Amount * Losing Probability)

Example 1:

You play a game where you are paid \$25 if you win, with a probability of 20% (0.2) and pay \$5 to play.

- Winning Gain = Prize - cost of playing: $\$25 - \$5 = \$20$
- Losing Amount = $-\$5$ (negative value)
- Winning Probability = 0.2
- Losing Probability = $1 - 0.2 = 0.8$

Expected Value = $(\$20 * 0.2) + (-\$5 * 0.8) = \$2 - \$4 = \mathbf{0}$

Interpretation:

This is a fair game: on average the player does not lose money.

Example 2:

You play a game where you are paid \$25 if you win, with a probability of 20% (0.2) and pay \$5 to play.

- Winning Gain = Prize - cost of playing: $\$25 - \$5 = \$20$
- Losing Amount = $-\$5$ (negative value)
- Winning Probability = 0.2
- Losing Probability = $1 - 0.2 = 0.8$

$$\text{Expected Value} = (\$20 * 0.2) + (-\$5 * 0.8) = \$4 - \$4 = \mathbf{-\$0}$$

Interpretation:

A negative expected value ($-\$0$ in this case) tell us that, on average, the gambler should expect to lose money when playing this game many times.

Notes:

- Expected value helps compare games and make informed decisions, not predicting individual results
- For complex games, consider using a table to organize calculations for each outcome.