

#### 4.5 Probability and Counting Rules Examples

The counting rules can be combined with the probability rules in this chapter to solve many types of probability problems.

1. A debit card pin number consists of four digits –selected from 0 to 9. The customer is allowed to repeat digits. What is the probability that someone who does not know the pin number in use, guesses the correct one?
2. The Florida Lottery consists of selecting six numbers from 1 through 53. What is the probability of winning the lottery?
3. Five-card Draw is one of the most classic poker games there is. What is the probability that someone is dealt five cards and they are all hearts?
4. Same situation as #3, but this time, find the probability that someone is dealt five cards and they are all of the same suit?
5. A box contains 24 transistors, 4 of which are defective. If 4 are sold at random, find the following probabilities:
  - a. Exactly 2 are defective.
  - b. None is defective.
  - c. All are defective.
  - d. At least 1 is defective.

## ANSWERS

1. A debit card pin number consists of four digits –selected from 0 to 9. The customer is allowed to repeat digits. What is the probability that someone who does not know the pin number in use, guesses the correct one?

Total number of possible outcomes:  $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10,000$

$$P = \frac{1}{10,000} = 0.0001$$

2. The Florida Lottery consists of selecting six numbers from 1 through 53. What is the probability of winning the lottery?

$$P(\text{winning}) = \frac{1}{{}^{53}C_6} = \frac{1}{22,957,480} = 0.0000000435 \dots$$

3. Five-card Draw is one of the most classic poker games there is. What is the probability that someone is dealt five cards and they are all hearts?

$$P(5 \text{ hearts}) = \frac{{}^{13}C_5}{{}^{52}C_5} = \frac{33}{66640} = 0.000495 \dots$$

4. Five-card Draw is one of the most classic poker games there is. What is the probability that someone is dealt five cards and they are all hearts?

=> P(5 hearts) or P(5 diamonds) or P(spades) or P(clubs)

P(5 hearts) + P(5 diamonds) + P(spades) + P(clubs)

$$\therefore 4 \cdot \frac{{}^{13}C_5}{{}^{52}C_5} = 4 \cdot \frac{33}{66640} = \frac{33}{16660} = 0.00198 \dots$$

5. A box contains 24 transistors, 4 of which are defective. If 4 are sold at random, find the following probabilities:

a. Exactly 2 are defective:

$$P(2 \text{ defective}) = \frac{{}^4C_2 \cdot {}^{20}C_2}{{}^{24}C_4} = \frac{1140}{10,626} \approx 0.107$$

b. None is defective:

$$P(\text{no defective}) = \frac{{}^{20}C_4 \cdot {}^4C_0}{{}^{24}C_4} = \frac{4845}{10,626} \approx 0.456$$

c. All are defective:

$$P(\text{all defective}) = \frac{{}^4C_4}{{}^{24}C_4} = \frac{1}{10,626} \approx 0.00009$$

d. At least 1 is defective:

$$P(\text{at least one defective}) = 1 - P(\text{none of them defective}) = 1 - 0.456 = 0.544$$