4.5 Counting rules and probability:

Counting rules and probability theory are closely interconnected. Counting rules help determine the total number of possible outcomes, while probability theory assigns probabilities to these outcomes. By combining the two, we can calculate the probability of specific events occurring, estimate frequencies, and analyze the likelihood of different scenarios.

1. Permutations and probability (distinct items). Example:
   Example 1: A group of students consists of Alice, Bob, Carol, Dave, and Eve. They are selected at random to present their project.
   a) What is the probability that they present in the following order: Eve, Alice, Dave, Carol and Bob?
   $Ans: \frac{1}{5!} = \frac{1}{120}$
   b) If instead, only three of them are selected for a presentation, what is the probability that the order of selection will be Alice, Bob, and Carol?
   $Ans: 1/_5P_3 = \frac{1}{60}$

2. **Example 2:** The letters in the word WORD are unscrambled by a software. For instance, in R programming the code is:
   word<-c("W", "O", "R", "D")
   result<-sample(word, size=4, replace=F)
   noquote(result)
   Note: run code at https://rdrr.io/snippets/
   Question: what is the probability that the result is WORD? $Ans: \frac{1}{4!} = \frac{1}{24}$

3. **Example 3:** Same scenario as the previous question, this time use the word STATISTICS.
   Code in R programming would be:
   word<-c("S", "T", "A", "T", "I", "S", "T" ,"I", "C" ,"S")
   result<-sample(word, size=10, replace=F)
   noquote(result)
   table(word) # it creates a table (count) of the letters
   Note: run code at https://rdrr.io/snippets/
   Question: what is the probability that the result is STATISTICS? $Ans: \frac{1}{\frac{10!}{2! \cdot 3! \cdot 3!}} = \frac{1}{50400}$

4. **Example 4:** Combinations and probability. Examples.
   A factory has a batch of 100 light bulbs, of which 10 are defective. If you randomly select 15 light bulbs for testing, what is the probability that exactly 3 of them are defective?

   $Ans: \frac{10C3 \cdot 90C12}{100C15} = 0.1297\dots$