4.4 Counting Rules:

Fundamental Counting Rule

In a sequence of n events in which the first one has k_1 possibilities ***and*** the second event has k_2 ***and*** the third has k_3, ***and*** so forth, the total number of possibilities of the sequence will be

$$k_1 * k_2 * k_3 \dots k_n$$

Note: In this case ***and*** means to multiply.

Question 8: In how many ways n distinct objects may be arranged?
When arranging n distinct objects, there are n choices for the first position in the sequence; $n-1$ for the second, $n-2$ for the third, etc. That is the product of all positive integers less than or equal to n. Mathematically, it is defined as: $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

Question 9: In how many ways n objects may be arranged if k of them are identical?
When arranging n objects, with k of them being identical, the number of distinct arrangements can be calculated using the formula for permutations with identical objects:

$$\frac{n!}{k!}$$

Where $n!$ is the factorial of the total number of objects and $k!$ is the factorial of the number of identical objects. This is because when we have identical objects, arranging them in different orders does not create distinct arrangements. Therefore, we divide the total number of permutations (where no objects are identical) by the factorial of the number of identical objects to eliminate the duplicate arrangements.

Question 10: In a laboratory, a biologist is arranging a sequence of DNA nucleotides for a genetic experiment. The sequence consists of 10 nucleotides, including: 3 adenine (A) nucleotides, 4 cytosine (C) nucleotides, 2 thymine (T) nucleotides and 1 guanine (G) nucleotide.

Ans: Number of arrangements $= \frac{10!}{3! \times 4! \times 2!} = 12600$

What if we arrange n objects taking r of them at a time?
The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taking r objects at a time. It is written as nPr, and the formula is:

$$nPr = \frac{n!}{(n-r)!}$$

What if we select k objects from a total of n objects without regard of order?

If we select k objects from a total of n objects without regard to order, we are dealing with combinations rather than permutations. In combinations, the order of selection does not matter. The formula to calculate the number of combinations is:

$$nCr = \frac{n!}{(n-r)!\,r!}$$

<u>Combinations and permutations on graphing Calculators:</u>

CASIO 9750: Main screen, OPTN, F6, then F3 for PROB

TI84: MATH, then PROB.