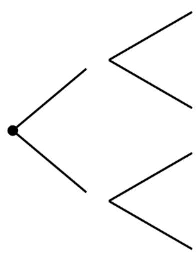
4.3 The Multiplication Rules and Conditional Probability:

The multiplication rule in probability is used to determine the probability that two or more independent events will occur together. The rule can be extended to dependent events as well, but the basic principle remains the same: it involves multiplying the probabilities of individual events to find the combined probability of their joint occurrence.

For independent events, the occurrence of one event does not affect the occurrence of another.
For example, tossing a coin is considered to produce independent events in probability because the outcome of one toss does not influence the outcome of any subsequent toss.

Question 1: toss a coin twice. What is the probability of obtaining Head and Head? Ans: **0.25**, Explain using the probability *tree:*



Question 2: The gender distribution of the nation has remained steady for several years, with women accounting for approximately 51.1 percent of the population since 2013. If you select five people at random in the US, what is the probability that they all are females? **Ans:** $(0.511)^5$ Explain

Question 3: Two people are selected simultaneously and at random from all people in the United States. Knowing that the probability of blood type O in the country is 0.44, what is the probability they are both blood type O?

In the previous examples the events are independent (the first outcome does not change the probability of the second outcome).

When two events are independent, the probability of both occurring is

$$P(A \text{ } and \text{ } B) = P(A) \, * \, P(B)$$

Dependent events:
When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such a way that the probability is changed, the events are said to be dependent events.

Examples of **dependent** events:
a. Drawing a card from a deck, not replacing it, and then drawing a second card.
b. Selecting a ball from an urn, not replacing it, and then selecting a second ball.
c. Smoking and developing lung cancer.

Question 4: Draw a card from a deck, do not replace it, and then draw a second card.
a) What is the probability that they are both hearts?
b) What is the probability that the first is a King and the second a Queen?
Question 5: There are 26 people at a meeting, 10 males and 16 females. They decide to select, at random, three people to be members of a committee. It happens that all three members of the committee are male. What is the probability o such outcome? Is it a rare event?

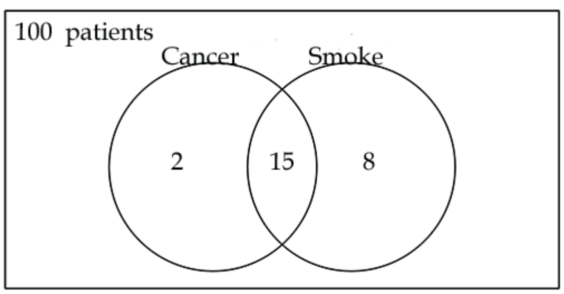
<u>The conditional probability of an event B in relationship to an event A is the probability that event B occurs after event A has already occurred. The notation for conditional probability is P(B|A).</u>

Question 6: A die is rolled. It is known that the outcome is a number greater than 3. What is the probability that the outcome is an even number?

Multiplication Rule for dependent events:
When two events are dependent, the probability of both occurring is

$$P(A\ and\ B) = P(A) * P(B|A)$$

Question 7: Given the information for 100 patients on the following Venn Diagram, find the probability of a patient having cancer given that he/she smokes.



$$P(C|S) = \frac{P(C \cap S)}{P(S)}$$

$Ans = 15/23$

Algebraically, the formula for Conditional Probability

$$P(B|A) = \frac{p(A\ and\ B)}{p(A)}$$

Is the result of dividing $P(A\ and\ B) = P(A) * P(B|A)$ by $P(A)$.

Probabilities for *At Least one*:

P (at least 1 of event E) = 1 – P (none of event E)

Question 8: A new drug for treating a certain condition has a 60% success rate. If two patients are given this drug independently, what is the probability that at least one of them will be successfully treated?

$P(at\ least\ 1\ success) = 1 - P(no\ success) = 1 - (0.40)^2 = 0.84$

Question 9: An adverse reaction to a drug occurs in 1 out of every 500 patients. A clinic has 1000 patients who have been prescribed this drug. What is the probability that at least one patient will experience an adverse reaction? May the Drs expect at least one adverse reaction?

P (at least 1 adverse reaction) = 1 – P (no adverse reaction among 1000 patients) = 1– $(0.998)^{1000}$= 0.86
Yes. Drs may expect at least one adverse reaction since the probability of such event is 86%