Notes on Probability

4.1 Basic Concepts:

Probability is the mathematical way of quantifying how likely something is to happen, based on the available evidence and assumptions. Probability is numerical measure that quantifies uncertainty.

Processes such as flipping a coin, rolling a die, or drawing a card from a deck are called probability experiments.

A **probability experiment** is a chance process that leads to well-defined results called outcomes.
An **outcome** is the result of a single trial of a probability experiment.
An **event** consists of a set of outcomes of a probability experiment.
A **sample space** is the set of all possible outcomes of a probability experiment.

Experiment	Sample space
Toss one coin	Head, tail
Roll a die	1, 2, 3, 4, 5, 6
Answer a true/false question	True, false
Toss two coins	Head-head, tail-tail, head-tail, tail-head

Equally likely events are events that have the same probability of occurring.

There are three basic interpretations of probability:

1. Classical probability
2. Empirical or relative frequency probability
3. Subjective probability

Formula for Classical Probability:

The probability of any event E is $P(E) = \dfrac{nE}{nS}$ where nE is number of outcomes of event E and nS is the total number of outcomes in the sample space.

Formula for Empirical Probability:

Given a frequency distribution, the probability of an event being is given by

$$pE = \frac{f}{n}$$

This probability is called empirical probability and is based on observation.

Law of Large Numbers: if a given probabilistic experiment is repeated a sufficient large number of times, the empirical probability approaches the theoretical probability.

Subjective Probability: The third type of probability is called subjective probability. Subjective probability uses a probability value based on an educated guess or estimate, employing opinions and inexact information.

Probability rules:

Probability Rule 1:
The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1.

Probability Rule 2:
If an event E cannot occur its probability is 0.

Probability Rule 3:
If an event E is certain, then the probability of E is 1.

Probability Rule 4:
The sum of the probabilities of all the outcomes in the sample space is 1.
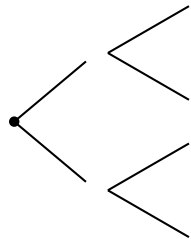
Probability Rule 5:

$$p(E) + p\bar{E} = 1$$

Where $p\bar{E}$ denotes the complement of an event E --- the set of outcomes in the sample space that are not included in the outcomes of event E.

Probability trees:
Finding the sample space for the gender of the children if a family has two children, using B for boy and G for girl.

The Standard Deck of Cards: