

Midpoint Formula, Distance Formula, and the Equation of a Circle

These three geometric concepts are fundamental in coordinate geometry. They are used to find the midpoint between two points, the distance between two points, and the equation of a circle in the coordinate plane.

1. Midpoint Formula

The midpoint of a segment connecting points $A(x_1, y_1)$ and $B(x_2, y_2)$ is the point halfway between them.

Formula:

$$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$$

Example:

Find the midpoint of $A(2, 3)$ and $B(6, 11)$:

$$M = ((2 + 6)/2, (3 + 11)/2) = (4, 7)$$

So the midpoint is $(4, 7)$.

2. Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the coordinate plane is found using the Pythagorean Theorem.

Formula:

$$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$$

Example:

Find the distance between $A(2, 3)$ and $B(6, 11)$:

$$d = \sqrt{((6 - 2)^2 + (11 - 3)^2)} = \sqrt{(16 + 64)} = \sqrt{80} = 4\sqrt{5} \approx 8.94$$

3. Equation of a Circle

A circle is the set of all points that are the same distance (the radius) from a fixed point (the center).

Standard Form:

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center and r is the radius.

Example 1:

Find the equation of a circle with center $(3, -2)$ and radius 5:

$$(x - 3)^2 + (y + 2)^2 = 5^2$$

$$(x - 3)^2 + (y + 2)^2 = 25$$

Example 2:

Given the circle equation $x^2 + y^2 - 6x + 8y + 9 = 0$, find the center and radius.

Step 1: Group x 's and y 's $\rightarrow (x^2 - 6x) + (y^2 + 8y) = -9$

Step 2: Complete the square for each group:

$$(x^2 - 6x + 9) + (y^2 + 8y + 16) = -9 + 9 + 16$$

$$(x - 3)^2 + (y + 4)^2 = 16$$

So, the center is $(3, -4)$ and the radius is 4.

Summary

- Midpoint Formula: Finds the point halfway between two coordinates.
- Distance Formula: Calculates the distance between two points.
- Equation of a Circle: Relates the center and radius of a circle using $(x - h)^2 + (y - k)^2 = r^2$.

These formulas are closely related through the Pythagorean Theorem and coordinate geometry concepts.