Finding the Inverse of a Function

The inverse of a function 'undoes' the action of the original function. If f takes x to y, then the inverse function, written f^{-1}, takes y back to x. Not all functions have inverses. A function must be one-to-one (each input gives a unique output and each output corresponds to one input) to have an inverse.

Steps to Find the Inverse

1. Write the function as $y = f(x)$.
2. Swap x and y.
3. Solve for y.
4. Rewrite y as $f^{-1}(x)$.

Examples

Example 1: Linear Function
$f(x) = 2x + 3$
Step 1: $y = 2x + 3$
Step 2: Swap x and y $\rightarrow x = 2y + 3$
Step 3: Solve for y $\rightarrow y = (x - 3)/2$
So, $f^{-1}(x) = (x - 3)/2$

Example 2: Quadratic Function (Restricted Domain)
$f(x) = x^2, x \geq 0$
Step 1: $y = x^2$
Step 2: Swap x and y $\rightarrow x = y^2$
Step 3: Solve for y $\rightarrow y = \sqrt{x}$ (since $x \geq 0$)
So, $f^{-1}(x) = \sqrt{x}$

Example 3: Rational Function
$f(x) = (x - 1)/(x + 2)$
Step 1: $y = (x - 1)/(x + 2)$
Step 2: Swap x and y $\rightarrow x = (y - 1)/(y + 2)$
Step 3: Multiply both sides $\rightarrow x(y + 2) = y - 1$

$xy + 2x = y - 1$

$xy - y = -2x - 1$

$y(x - 1) = -2x - 1$

$y = (-2x - 1)/(x - 1)$

So, $f^{-1}(x) = (-2x - 1)/(x - 1)$

Summary

- The inverse function reverses the input-output rule of the original function.
- To find an inverse: swap x and y, then solve for y.
- Functions must be one-to-one to have inverses.
- Always check your answer by verifying that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.