

Finding the Domain of a Function

The domain of a function is the set of all possible input values (x-values) for which the function is defined. Determining the domain depends on the type of function involved. Below are the main cases for polynomial, rational, square root, and cubic root functions.

1. Polynomial Functions

Form: $f(x) = ax^n + bx^{n-1} + \dots + c$

Example: $f(x) = 2x^3 - 5x + 7$

Domain: All real numbers (\mathbb{R})

Reason: Polynomials are defined for every real x; there are no restrictions such as denominators or square roots.

Example: $f(x) = 3x^2 - 4x + 1 \rightarrow$ Domain: $(-\infty, \infty)$

2. Rational Functions

Form: $f(x) = P(x) / Q(x)$, where P and Q are polynomials.

Restriction: The denominator $Q(x)$ cannot be zero.

To find the domain:

1. Set $Q(x) = 0$.
2. Exclude the values of x that make $Q(x) = 0$.

Example: $f(x) = (x + 2) / (x^2 - 9)$

Denominator = $x^2 - 9 = (x - 3)(x + 3)$

$Q(x) = 0$ when $x = 3$ or $x = -3$

Domain: All real numbers except $x = -3, 3 \rightarrow (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

3. Square Root Functions

Form: $f(x) = \sqrt{x}$ or $f(x) = \sqrt{g(x)}$

Restriction: The expression under the square root (radicand) must be ≥ 0 .

To find the domain:

1. Set the radicand ≥ 0 .
2. Solve for x.

Example: $f(x) = \sqrt{x - 5}$

$x - 5 \geq 0 \rightarrow x \geq 5$

Domain: $[5, \infty)$

4. Cubic Root Functions

Form: $f(x) = \sqrt[3]{x}$

Domain: All real numbers (\mathbb{R})

Reason: Cubic roots are defined for both positive and negative numbers.

Example: $f(x) = \sqrt[3]{x - 7}$

Domain: $(-\infty, \infty)$

Summary Table

Type of Function	General Form	Domain Rule	Example	Domain
Polynomial	$P(x)$	All real numbers	$2x^3 - 5x + 7$	$(-\infty, \infty)$
Rational	$P(x)/Q(x)$	$Q(x) \neq 0$	$\frac{x+2}{x^2-9}$	$x \neq \pm 3$
Square Root	$\sqrt{g(x)}$	$g(x) \geq 0$	$\sqrt{x-5}$	$x \geq 5$
Cubic Root	$\sqrt[3]{g(x)}$	All real numbers	$\sqrt[3]{x-7}$	$(-\infty, \infty)$

Key Idea:

To find the domain, identify any operations that could make the function undefined (division by zero or even roots of negative numbers) and exclude those values.