## Identifying Even and Odd Functions

In college algebra, functions are often classified as even, odd, or neither based on their symmetry properties. This classification helps us understand the graph of a function more easily.

### Definitions

A function $f(x)$ is **EVEN** if: $f(-x) = f(x)$ for all $x$ in the domain.
Graphically, even functions are symmetric about the $y - axis$.

A function $f(x)$ is **ODD** if: $f(-x) = -f(x)$ for all x in the domain.
Graphically, odd functions are symmetric about the origin.

### How to Prove

To determine whether a function is even, odd, or neither, substitute -x into the function and simplify:

- If $f(-x) = f(x)$, the function is even.
- If $f(-x) = -f(x)$, the function is odd.
- If neither condition holds, the function is neither even nor odd.

### Examples

1. Even Function: $f(x) = x^2 + 1$

$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x)$.
Notice that in a polynomial where all powers of $x$ are even, the function is even. Assume that the constant term, 1 in this example, includes $x^0$ (zero es even)

Graph: Symmetric about the $y - axis$.

2. Even Function: $f(x) = x^4 - x^2 + 7$

$f(-x) = (-x)^4 - (-x)^2 + 7 = x^4 - x^2 + 7 = f(x)$

Graph: Symmetric about the $y - axis$.

3. Odd Function: $f(x) = x^3 + 10x$

$f(-x) = (-x)^3 + 10(-x) = -x^3 - 10x = -f(x)$.

Notice that all powers of $x$ are odd.

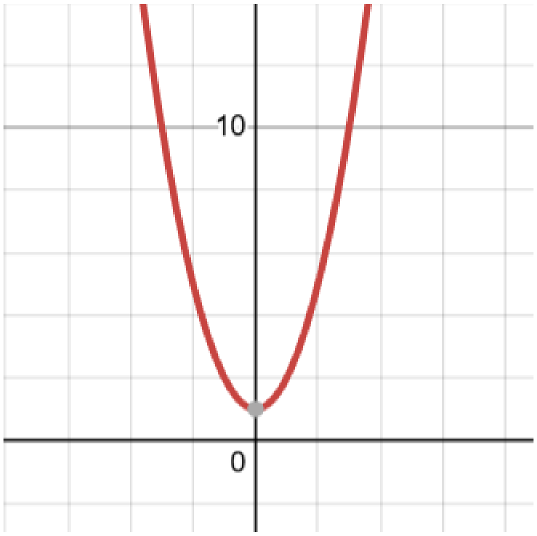
Graph: Symmetric about the origin.

4. Neither: $f(x) = x^2 + x - 6$

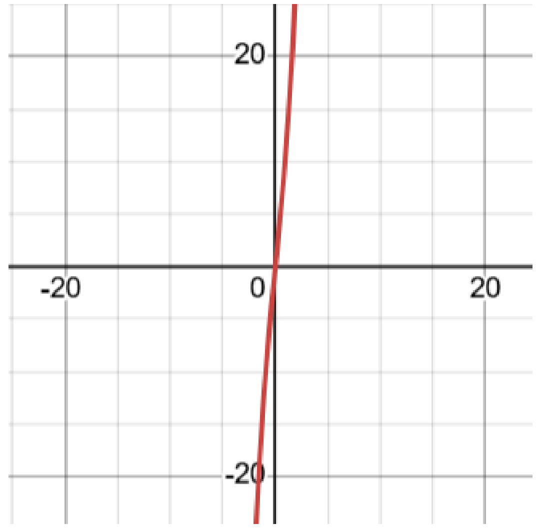
$f(-x) = (-x)^2 + (-x) - 6 = x^2 - x - 6 \neq f(x) \neq -f(x)$

In this last example, we have a mix of even and odd powers of $x$. Neither even nor odd.

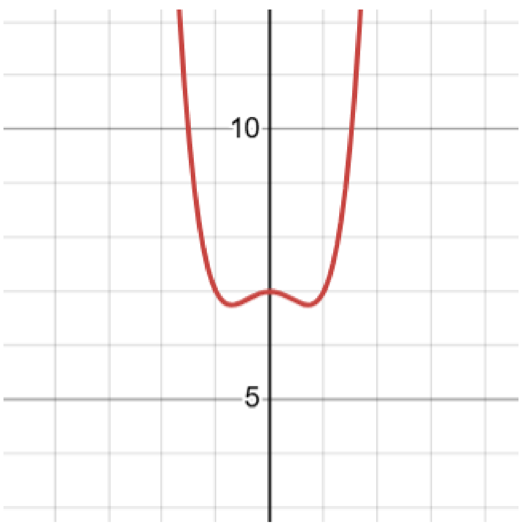
**Graphs** on page 2:



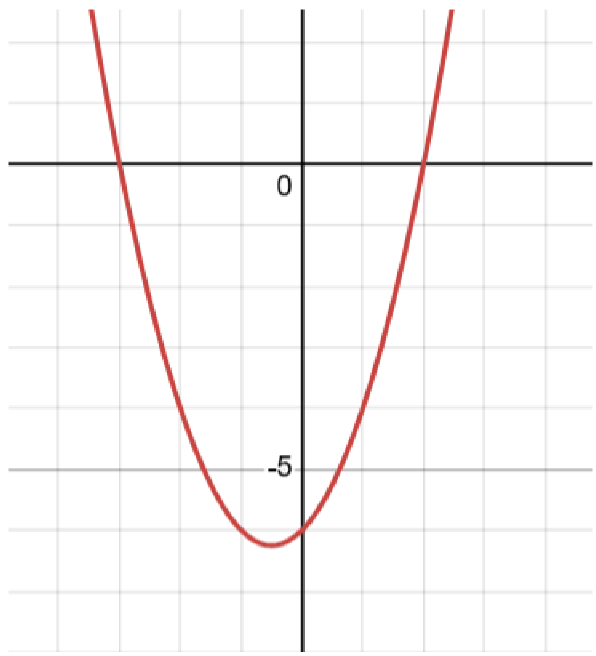

$f(x) = x^2 + 1$




$f(x) = x^3 + 10x$




$f(x) = x^4 - x^2 + 7$




$f(x) = x^2 + x - 6$

https://mystatclass.com/mac1105w.html