

The Difference Quotient

In college algebra and precalculus, the difference quotient is a formula that represents the average rate of change of a function over an interval. It is the foundation for the concept of the derivative in calculus.

Meaning

The difference quotient measures how much a function changes between two points. It is written as:

$$\frac{f(x + h) - f(x)}{h}$$

Here, h is a small change in x . The numerator, $f(x + h) - f(x)$, represents the change in the output values of the function, while dividing by h gives the rate of change per unit of x .

Rationale

The difference quotient is essentially the slope formula for a secant line connecting two points on the graph of a function: $(x, f(x))$ and $(x + h, f(x + h))$. In calculus we say that, as h approaches zero, this slope becomes the *instantaneous rate of change*, also known as the **derivative**.

Examples

1. Linear Function: $f(x) = 2x + 3$

$$f(x + h) = 2(x + h) + 3 = 2x + 2h + 3$$

$$f(x + h) - f(x) = (2x + 2h + 3) - (2x + 3) = 2h$$

$$\text{Difference Quotient} = (2h)/h = 2$$

This shows that the slope of a linear function is constant, so the difference quotient is always equal to the slope.

2. Quadratic Function: $f(x) = x^2 + 1$

$$f(x + h) = (x + h)^2 + 1 = x^2 + 2xh + h^2 + 1$$

$$f(x + h) - f(x) = (x^2 + 2xh + h^2 + 1) - (x^2 + 1) = 2xh + h^2$$

$$\text{The difference quotient} = (2xh + h^2)/h = 2x + h$$

Unlike the linear case, the result depends on x and h . As h gets smaller, the value approaches $2x$, the slope at that point.