Composite of Functions and Operations with Functions

In algebra, we often combine functions in different ways to create new functions. Two common approaches are: (1) performing arithmetic operations with functions, and (2) composing functions.

Operations with Functions

If f(x) and g(x) are two functions, we can define new functions as follows:

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(f \cdot g)(x) = f(x) \cdot g(x)$
- $(f / g)(x) = f(x) / g(x)$, provided $g(x) \neq 0$

Example:
Let $f(x) = 2x + 1$ and $g(x) = x^2$
$(f + g)(x) = (2x + 1) + (x^2) = x^2 + 2x + 1$
$(f \cdot g)(x) = (2x + 1)(x^2) = 2x^3 + x^2$

Composite of Functions

The composite function (f ∘ g)(x) means f(g(x)), where we apply g first, then f.

Example:
Let $f(x) = 3x - 4$ and $g(x) = x^2 + 1$
$(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = 3(x^2 + 1) - 4 = 3x^2 - 1$
$(g \circ f)(x) = g(f(x)) = g(3x - 4) = (3x - 4)^2 + 1 = 9x^2 - 24x + 17$

Summary

- Operations with functions allow us to add, subtract, multiply, or divide functions.
- Composite functions involve substituting one function into another.
- Order matters in composition: $f \circ g$ is generally different from $g \circ f$.

https://mystatclass.com/mac1105w.html