Solving Quadratic Equations by Completing the Square

The completing the square method is a systematic way to solve quadratic equations. It transforms a quadratic expression into a perfect square trinomial, making it easier to solve for the variable.

1. Case: $x^2 + bx + c = 0$

Steps:
1. Start with the quadratic equation: $x^2 + bx + c = 0$
2. Move the constant term to the other side: $x^2 + bx = -c$
3. Add $(b/2)^2$ to both sides to complete the square:
 $x^2 + bx + (b/2)^2 = -c + (b/2)^2$
4. Rewrite the left side as a perfect square:
 $(x + b/2)^2 = (b^2/4) - c$
5. Take the square root of both sides:

$$x + b/2 = \pm\sqrt{\left((b^2/4) - c\right)}$$

6. Solve for x:

$$x = -b/2 \pm \sqrt{\left((b^2/4) - c\right)}$$

2. Case: $ax^2 + bx + c = 0$, with $a \neq 1$

Steps:
1. Start with the quadratic equation: $ax^2 + bx + c = 0$
2. Divide through by a (to make the coefficient of x^2 equal to 1):
$x^2 + (b/a)x + (c/a) = 0$
3. Move the constant term to the other side:
 $x^2 + (b/a)x = -c/a$
4. Add $(b/2a)^2$ to both sides:
$x^2 + (b/a)x + (b/2a)^2 = -c/a + (b^2/4a^2)$
5. Rewrite the left side as a perfect square:
$(x + b/2a)^2 = (b^2 - 4ac)/(4a^2)$
6. Take the square root of both sides:
 $x + b/2a = \pm\sqrt{(b^2 - 4ac)}/(2a)$
7. Solve for x:

$$x = \left(-b \pm \sqrt{(b^2 - 4ac)}\right)/(2a)$$

Conclusion: Completing the square not only provides a method to solve quadratic equations, but it also leads directly to the quadratic formula:

$$x = \left(-b \pm \sqrt{(b^2 - 4ac)}\right)/(2a)$$