

Absolute Value Equations

1. Definition of Absolute Value

The absolute value of a real number x , written $|x|$, represents the distance from x to 0 on the number line.

2. Basic Principle for Solving Equations

An equation of the form $|x| = a$ (where $a \geq 0$) has two solutions:

$$x = a \text{ or } x = -a$$

If $a < 0$, the equation $|x| = a$ has no solution.

3. Steps to Solve Absolute Value Equations

- Isolate the absolute value expression.
- Write two equations: one for the positive case and one for the negative case.
- Solve each equation.
- Check for extraneous solutions (sometimes restrictions may apply).

4. Examples

Example 1: Solve $|x| = 5$. It is read: what are the numbers whose distance from zero is 5?
Solutions: $x = 5$ or $x = -5$

Example 2: Solve $|x - 3| = 7$. It is read: what are the numbers whose distance from 3 is 7?

Case 1: $x - 3 = 7 \rightarrow x = 10$

Case 2: $x - 3 = -7 \rightarrow x = -4$

Solutions: $x = 10, -4$

Example 3: Solve $|x + 1| = -3$. It would be read, what are the numbers whose distance from -1 is -3 ?

Since no distance is negative, there is no solution.